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# A multiple-source air quality control model achieving a standard, defined by a vector-valued function

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**Abstract** Earlier papers by Gustafson, Kortanek, Sweigart and others describe models for controlling air pollution, consisting of chemically inert pollutants like sulphur dioxide. It was assumed that the concentration contributions from the sources added up at each receptor point. The goal was to achieve acceptable air quality for each receptor point, generally defined by the annual mean concentration of the pollutant under study. The set of polluting sources was split in $n$ source-classes, where the sources in each class were regulated in the same way and independently of the other source-classes. One such class could be motor vehicles which are required to use the same quality of fuel, since it is of course not possible to regulate the pollution output from each vehicle separately. The idea was to determine the relation between the strength of each source-class and its contribution to the annual mean concentration at each receptor point in the air quality control area. Then one calculates how the strengths of sources need to be reduced to achieve the desired air quality. Generally, there are many reductions policies which achieve this goal and one seeks to calculate the policy which achieves this at the lowest total regulation cost. This model requires large amounts of data, since one needs to have lists of all source-classes as well as meteorological information in order to calculate the contributions to the mean concentration. Here we propose to discretise the set of weather states as well. Each weather situation is defined by meteorological parameters like wind speed, wind direction, mixing height and so on. The idea is to represent the set of all possible weather situations in the control area by $k$ points $w_1, \ldots, w_k$ in the meteorological parameter space with associated probabilities $p_1, \ldots, p_k$. Thus the climate in the air quality control area is represented by this discrete probability distribution. Next we introduce standards for each weather state defined by the functions $w_1, \ldots, w_k$. These standards are determined such that the permissible pollution has a desirable distribution, e.g. such that the probability of very high concentrations is low. It is assumed that it is known which weather states give the highest pollution concentrations. Hence we get $k$ constraints at each receptor point and we may calculate, using semi-infinite programming, the reductions policy which satisfies these constraints at minimum control costs. Similar models may be developed for water pollution problems.

**Keywords** Source-class, air quality, mathematical model, optimal reductions policy.

# 1 INTRODUCTION

## 1.1 Pollutants

We consider the task of maintaining a satisfactory air quality in an air quality control area, e.g. a city. Several pollutants may be observed from measurements, e.g. sulphur dioxide $SO_2$ and carbon monoxide. For further details see e.g. (Air Trends, 2008). The goal is to achieve that the concentrations of all identified pollutants stay below legally defined tolerances throughout the control are at all times. This is achieved by regulating the emissions from identified sources. We will describe how to construct a mathematical model for this purpose.

## 1.2 Air quality standards

We will limit our discussion to the chemically inert pollutant $SO_2$, which has been studied in many contexts. See (Fahlander et al., 1974) and (Stavins, 2005). We assume that there is a standard requiring that the annual mean concentration is not allowed to surpass a given number $b$ at any point in the control area. In addition, there may be standards which must not be surpassed for more than a time period of predetermined length.

## 1.3 Source inventory and cost functions

One needs a list of the strengths of all sources which contribute to the pollution. The idea is to impose constraints on the emissions of these sources. Some sources cannot be regulated at all, maybe because they are outside the control area, possible in another country, and their contributions make up what here will be termed the background pollution. The remaining sources will be split into $n$ source-classes, where the polluters in each class are to be regulated in the same way. Hence we have $n$ control variables, namely the fractions $E_r$ with which each source-class $r$ is required to cut back its emissions. The associated cost function will be denoted $c_r(E_r)$ and it is assumed to be differentiable, convex and nondecreasing.

## 1.4 Impact

Each source-class, in the future termed pollutor, gives a contribution to the concentration in the air at each point on the ground (henceforth called receptor point). It is assumed that the contributions from the pollutors add up and that the contribution from each pollutor is proportional to its strength. Hence we get a linear constraint for each receptor point and these are infinitely many. Thus the techniques of semi-infinite programming will be required. See (Glashoff–Gustafson, 1983), p 18 and also (Goberna-López, 1997) as well as (Reemtsen-Rückmann, 1998). We note that the observed pollution at a given receptor point depends also on the weather situation which is specified by meteorological parameters such as wind speed and direction and index for stability of the atmosphere. An overview of models which connect the properties of a pollutor and the weather situation to its contribution to the pollutant concentration in the control area is given in (Moussiopoulos, N., 1996). For a long-term model we need information about the climate, which we specify by listing the typical weather states defined by the meteorological parameter vectors $w_1, \ldots, w_k$ as well as their frequencies $p_1, \ldots, p_k$. Thus we determine the transfer functions for each state and obtain the annual mean by averaging using the weights $p_1, \ldots, p_k$.

## 2 MATHEMATICAL MODEL FOR AIR QUALITY MANAGEMENT

We use the notations and definitions from Section 1. The control area is represented by the two-dimensional set $S$. Pollutor number $r, r = 1, \ldots, n$ is assumed to have reduced its emission with the fraction $E_r$, where

$$0 \leq E_r \leq e_r \leq 1, r = 1, \ldots, n, \tag{1}$$

where $e_r$ is a bound for the technically possible reduction.

For weather situation $i,\ i = 1, \ldots, k$ with the frequency $p_i$ and the pollutor $r,\ r = 1, \ldots, n$ we have calculated the transfer function

$$u_r^i(s),$$

which describes the impact of pollutor $r$ at the receptor point $s \in S$ and the transfer function of the background emissions is

$$u_0^i(s).$$

After reduction we have the remaining pollutant concentration

$$\sum_{r=1}^{n}(1 - E_r)u_r^i(s) + u_0^i(s),\ s \in S.$$

Since the standard for weather state number $i$ is $b_i$ we have for a feasible abatement policy

$$\sum_{r=1}^{n} E_r u_r^i(s) \geq \sum_{r=0}^{n} u_r^i(s) - b_i, s \in S,\ i = 1, \ldots, k, \tag{2}$$

$$0 \leq E_r \leq e_r,\ r = 1, \ldots, n. \tag{3}$$

Hence $k$ sets of inequalities of the type of (2) must be satisfied for a feasible reductions policy. We note that (2) may be interpreted as the condition that at least the difference between the original pollution and the standards for all the weather conditions must be removed. We next discuss the relation with standards for the annual mean pollution concentration. The transfer functions $u_r$ and standard $b$ for the annual mean concentration are given by

$$u_r(s) = \sum_{i=1}^{k} p_i u_r^i(s),\ r = 0, \ldots, n, \tag{4}$$

$$b = \sum_{i=1}^{k} p_i b_i. \tag{5}$$

If $E$ defines a feasible reduction policy, satisfying (2) and (3) we have, since

$$p_i \geq 0,\ \sum_{i=1}^{k} p_i = 1,$$

$$\sum_{r=1}^{n} E_r u_r(s) \geq \sum_{r=0}^{n} u_r(s) - b \geq 0. \tag{6}$$

We observe that if $E$ satisfies (2) and (3), then it also satisfies (3) and (6), but the converse does not need to be true. The data acquisition effort is about the same for (2), (3) as for (3), (6) but the former sets of conditions gives more flexibility is setting standards for handling in advance extremely adverse conditions associated with unfavourable weather conditions which will occur with a known probability.

Many vectors $E \in R^n$ may define a feasible reductions policy. In Subsection 1.3 we introduced the control costs $c_r(E_r)$. In the next Section we will show how to minimise the combined control costs

$$\sum_{r=1}^{n} c_r(E_r),$$

and it is of interest to compare this minimal cost with the cost of the reductions policy actually chosen in a practical situation.

# 3 CALCULATING AN OPTIMAL REDUCTIONS POLICY

In this Section we will need the following

**Theorem 1** *Let $S$ be a compact set, $C(S)$ the Banach space of continuous functions defined on $S$ and equipped with the maximum norm. Let $L$ be a bounded linear functional on $C(S)$. Then $L$ may be represented by the Stieltjes integral*

$$L(f) = \int_S f(s) d\alpha(s),$$

*with*

$$||L|| = \int_S |d\alpha(s)|.$$

**Remark 1** *The linear functional $L$ is termed positive if $f(s) \geq 0,\ s \in S$ implies $L(f) \geq 0$.*

We give

**Example 1** *Let $f \in C(S)$ and put*

$$L(f) = \sum_{i=1}^{q} x_i f(s_i),\ s_i \in S.$$

*Then*

$$||L|| = \sum_{i=1}^{q} |x_i|,$$

*and the Stieltjes integral is represented by a sum.*

To determine an optimal reductions policy we consider the optimisation problem

$$\min_{E \in R^n} \sum_{r=1}^{n} c_r(E_r),$$

subject to the constraints

$$\sum_{r=1}^{n} E_r u_r^i(s) \geq \sum_{r=0}^{n} u_r^i(s) - b_i,\ s \in S,$$

$$E_r \geq 0,\ r = 1, \ldots, n,$$

$$-E_r \geq -e_r\ r = 1, \ldots, n.$$

This problem may be written

$$\min_{E \in R^n} \sum_{r=1}^{n} c_r(E_r),$$

subject to the constraints

$$\sum_{r=0}^{n} u_r^i(s) - b_i - \sum_{r=1}^{n} E_r u_r^i(s) \leq 0,\ s \in S, i = 1, \ldots, k,$$

$$-E_r \leq 0,\ r = 1, \ldots, n,$$

$$E_r - e_r \leq 0,\ r = 1, \ldots, n.$$

We note that there is a feasible $E \in R^n$ which gives strict inequalities. (Slater's condition). In addition, the cost function has a finite minimum. Therefore we may use the theorem on Lagrangian duality on p224 in (Luenberger, 1969) to determine an optimal solution. We start by defining the Lagrangian dual to our optimisation problem:

$$\max \min_{E \in R^n} \sum_{r=1}^{n} c_r(E_r) + \sum_{i=1}^{k} \int_S d\alpha_i(s)\Delta_i(s) - \sum_{r=1}^{n} \lambda_r E_r + \sum_{r=1}^{n} \mu_r(E_r - e_r).$$

Here we have

$$\Delta_i(s) = \sum_{r=0}^{n} u_r^i(s) - b_i - \sum_{r=1}^{n} E_r u_r^i(s),$$

and the maximisation is carried out over

$$d\alpha(s) \geq 0, \lambda_r \geq 0, \mu_r \geq 0, \; r = 1, \ldots, n.$$

We arrive at the optimality conditions:

$$\sum_{i=1}^{n} \int_S d\alpha_i(s) u_r^i(s) + \lambda_r - \mu_r = c_r'(E_r), \; r = 1, \ldots, n,$$

$$\lambda_r E_r = 0, \; r = 1, \ldots, n,$$

$$\mu_r(E_r - e_r) = 0,$$

$$d\alpha_i(s_{j_i})\Delta_i(s_{j_i}) = 0, j = 1, \ldots, q_i.$$

The functions $\Delta_i$ has a local maximum at $s_{j_i}$ if $d\alpha(s_{j_i}) > 0$. The optimal measures are given by the finite sums

$$\int_S d\alpha_i(s) g(s) = \sum_{j=1}^{q_i} g(s_{j_i}),$$

for any continuous function $g(s)$

# 4 IMPLEMENTING A FEASIBLE REDUCTIONS POLICY

## 4.1 Introduction

The optimal reductions policy described in Section 3 may be difficult to implement in a practical context. However, it may be used as a bench-mark with which one may compare the efficiency of other policies which may be more palatable from a political point of view and also guarantee that an acceptable air quality is obtained, albeit at a higher combined control cost. We discuss here some alternative strategies.

## 4.2 Proportional reduction

All pollutors are required to cut back their emissions with the same fraction. The latter is determined to be sufficiently large to guarantee compliance with standards. This strategy will penalise pollutors which at the start already have low emissions and will come cheap for those who start with high emissions.

## 4.3 Pollution tax

Many countries have introduced taxes on pollutants emitted into the environment. In the ideal situation all pollutors reduce their emissions until the marginal cost of cutting back emissions equals the tax. Thus raising the tax will lead to a cleaner environment and higher combined control costs.

## 4.4 Cooperative schemes

It may be such that the cost for cutting back emissions may be different for different pollutors. Then it would be cheaper for some pollutors to pay other sources for cleaning up. (Carbone et al., 1978) describe a scheme where the pollutors have the collective responsibility of implementing a reductions strategy which satisfies the standard. If they cooperate, they could choose an optimal policy which gives the minimal combined cost.

## 4.5 Trading emission allowances

(Stavins, 2005) describes systems for $SO_2$ allowances trading in the US. Their implementation has resulted in major improvements in the form of a cleaner environment and lower combined abatement costs. For years the Norwegian government has pushed for the implementation of an international system for trading with carbon dioxide emission allowances. However, in a recent article in the Norwegian business daily Dagens Næringsliv it is claimed that this trade has not resulted in any improvement in the global situation. Both Norway and the rest of the world combined release more carbon dioxide into the atmosphere than a few years ago.

### 4.6 CONCLUSIONS

We have described air quality control models for calculating cost-efficient abatement policies. In comparison to earlier related papers we have extended the computational scheme to convex, not just linear, cost functions. Without needing more data than before, e.g. in (Fahlander et al., 1974) we have shown how to introduce different standards for different weather states, not just guaranteeing that the annual mean pollution concentration is acceptable. Thus we may calculate a policy which also limits the pollutant concentration in adverse situations.

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