18th World IMACS / MODSIM Congress, Cairns, Australia 13-17 July 2009
http://mssanz.org.au/modsim09

Modelling Hong Kong stock index by student *t*-mixture autoregressive model

Wong, C.S. [1]

[1] *Department of Finance, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong*
Email: albertw@baf.msmail.cuhk.edu.hk

Abstract: It is well known that most of the financial and economic time series possess several stylized properties which include time-varying means and variances (Granger and Teräsvirta, 1993; Franses and van Dijk, 2000; and Tsay, 2002). Moreover, there is strong evidence that most of the financial return series are not normally distributed. They usually exhibit excess kurtosis which means that there are more probability mass in the tails of the marginal or conditional distribution.

Linear time series models such as autoregressive and moving average models assume that the conditional and unconditional distributions of the time series are Gaussian with time-independent variances. Nonlinear time series models such as threshold (Tong, 1990) and GARCH models relax the constant variance assumption but still ally with the assumption that the financial returns are conditionally normal distributed.

Mixture-typed time series models, introduced by Wong and Li (2000), are potentially useful in modelling of financial returns. The models are able to capture serial correlations; time-varying means and volatilities; and the shape of the conditional distributions can be time varied from short-tailed to long-tailed, or from unimodal to multi-modal. However, as these models ally with the assumption that return series in each component is conditionally Gaussian, it may underestimate the occurrence of extreme financial events, such as market crashes.

Wong et al. (2009) extend the mixture-typed time series models by replacing the Gaussian assumption with the Student *t* distribution. The Student *t*-mixture autoregressive (TMAR) model consists of a mixture of *g* autoregressive components with Student *t* error distributions. As the tails of Student *t* distribution can be much heavier than that of normal distribution, it would be able to accommodate some aberrant returns occasionally observed in financial markets. The use of *t* distributed errors in each component of the model allows for conditional leptokurtic distribution which could account for the commonly observed unconditional kurtosis in the financial data. They successfully apply the class of models to the US 3-year Treasury Constant Maturity Rate. Comparison studies indicate that TMAR model has better performance of interval prediction over other commonly used time series models.

It is well known that interest rates series and stock indices series behave differently. For example, stock indices behave like a random walk while interest rates evolve with mean-reversion. Hence, the observations for interest rates data in Wong et al. (2009) cannot be directly generalized for stock indices data. In this paper, we apply the class of TMAR models to the return series of Hong Kong Heng Sang Index. We find that the TMAR model also has better performance of interval prediction in the case of stock index data.

Keywords: *Hong Kong Hang Seng Index, non-linear time series models*

1. INTRODUCTION

Strong evidence has indicated that most of the financial return series are not normally distributed. They usually exhibit excess kurtosis which means that there are more probability mass in the tails of the marginal or conditional distributions. Since most of the time series models ally with the assumption that return series is marginally and/or conditionally Gaussian, it may underestimate the occurrence of extreme financial events, such as market crashes.

The mixture autoregressive (MAR) model introduced by Wong and Li (2000) consists of a mixture of g autoregressive (AR) components. There are several properties which make the MAR process potentially useful in modeling nonlinear time series. The MAR model is able to capture serial correlations; time-varying means and volatilities; and the shape of the conditional distributions can be time varied from short-tailed to long-tailed, or from unimodal to multi-modal. However, its alliance with the assumption that the return series in each of the autoregressive component is conditionally Gaussian may make it unable to estimate the occurrence of extreme financial events accurately.

Wong et al. (2009) extend the MAR model by replacing the Gaussian assumption with the Student t distribution. The Student t distribution provides heavier tails than normal distribution and would be able to accommodate some aberrant returns occasionally observed in financial markets. The degrees of freedom in the Student t distributions are parameters which can be used to adjust the degree of thickness of the conditional distributions according to the actually observed data. As the degrees of freedom in a Student t distribution approaches infinity, the distribution approaches normal. Hence the MAR model is a limiting case of the Student t-mixture autoregressive (TMAR) model.

In this paper, we apply the class of TMAR models to the return series of Hong Kong Heng Seng Index. Results from comparison studies are reported which indicate that TMAR models provides a better description of conditional distributions of the return series than other models allied with the Gaussian assumption.

The paper proceeds as follows. Section 2 provides a brief review of TMAR modelling. Section 3 presents the empirical results. The discussion and conclusion follow in the final section.

2. THE TMAR MODEL

2.1. Model specification

Wong et al. (2009) introduce the class of Student t-mixture autoregressive models. This model consists of a mixture of g autoregressive components with Student t distributed innovations. A TMAR(g; p_1, p_2, …, p_g) model is defined by

$$F\left(y_t \middle| \mathcal{F}_{t-1}\right) = \sum_{k=1}^{g} \alpha_k F_{\nu_k}\left(\frac{y_t - \phi_{k0} - \phi_{k1} y_{t-1} - \ldots - \phi_{kp_k} y_{t-p_k}}{\sigma_k}\right).$$

Here, $F\left(y_t \middle| \mathcal{F}_{t-1}\right)$ is the conditional cumulative distribution function of Y_t given the past information, evaluated at y_t; $\mathcal{F}_t$ is the information set up to time t; $F_{\nu_k}(\cdot)$ is the cumulative distribution function of the standardized Student t distribution with ν_k degrees of freedom; and mixing proportions $\alpha_1 + \ldots + \alpha_g = 1$ with $\alpha_k > 0$, for $k = 1, \ldots, g$. It should be noted that the probability distribution function (p.d.f.) of a standardized (i.e. normalized to have unit variance) Student t distribution with ν degrees of freedom is given by

$$f_\nu(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\,\Gamma\left(\frac{\nu}{2}\right)}\left\{1 + \frac{x^2}{\nu-2}\right\}^{-\frac{\nu+1}{2}},$$

where $2 < \nu < \infty$, and $\Gamma(\cdot)$ is the gamma function.

The first and second conditional moments of TMAR model are same as that of MAR model. The conditional expectation of y_t is $E\left(y_t \middle| \mathcal{F}_{t-1}\right) = \sum_{k=1}^{g} \alpha_k \left(\phi_{k0} + \phi_{k1} y_{t-1} + \ldots + \phi_{kp_k} y_{t-p_k}\right) = \sum_{k=1}^{g} \alpha_k \mu_{k,t}$. The shapes of the conditional distributions may change from unimodal to multi-modal. Therefore, the conditional expectation of y_t may not be the best predictor of the future values. The conditional variance of y_t is $\operatorname{var}\left(y_t \middle| \mathcal{F}_{t-1}\right) = \sum_{k=1}^{g} \alpha_k \sigma_k^2 + \sum_{k=1}^{g} \alpha_k \mu_{k,t}^2 - \left(\sum_{k=1}^{g} \alpha_k \mu_{k,t}\right)^2$. The first term summarizes the conditional variance on the past "errors". The second and third terms model the change of the conditional variance due to the difference in the conditional means of the components.

The first-order and second-order stationarity conditions for TMAR model are exactly the same as that for MAR model. Theorem 1 gives the condition of first-order stationarity for a general TMAR model while Theorem 2 gives the condition for second-order stationarity for a TMAR(g; 1, …, 1) model. The proofs of all theorems are given in a longer version of Wong et al. (2009). Let $p = \max(p_1, \ldots, p_g)$.

Theorem 1. A necessary and sufficient conditions for the process Y_t to be first-order stationary is that the roots $z_1, \ldots, z_p$ of the equation $1-\sum_{i=1}^{p}\left(\sum_{k=1}^{g}\alpha_k\phi_{ki}\right)z^{-i}=0$ all lies inside the unit circle, where $\phi_{ki} = 0$ for $i > p_k$.

Theorem 2. Suppose that the process Y_t following a TMAR(g; 1, …, 1) model is first-order stationary. A necessary and sufficient condition of the process to be second-order stationary is $\alpha_1\phi_{11}^2+\alpha_2\phi_{21}^2+\ldots+\alpha_g\phi_{g1}^2<1$.

It is possible that a mixture of an explosive AR component and a stationary AR component results in an overall stationary process. For illustration, we consider the following TMAR(2; 1, 1) model, $F\left(y_t|\mathcal{F}_{t-1}\right)=0.75F_{10}\left\{\left(y_t-0.5y_{t-1}\right)/5.0\right\}+0.25F_{20}\left\{\left(y_t-1.4y_{t-1}\right)/1.0\right\}$. The first component in the above model is stationary while the second component is non-stationary. The first-order and second-order stationarity conditions for the TMAR(2; 1, 1) model are $\left|\alpha_1\phi_{11}+\alpha_2\phi_{21}\right|<1$ and $\alpha_1\phi_{11}^2+\alpha_2\phi_{21}^2<1$, respectively. It is easy to verify that this process Y_t satisfies the first-order and second-order stationarity conditions and results in an overall stationary process.

The major advantage of the TMAR model is its ability to model the tails of the conditional distribution. Owing to the Gaussian assumption, MAR models are often unable to explain the leptokurtosis in the data. On the other hand, the TMAR model is more flexible in modelling the tails due to the Student *t* distribution assumption on each component series of innovations. The degrees of freedom of the Student *t* distribution in the *k*th component (ν_k) can be fixed in advance or inferred from observations. The shape and tails of the conditional distribution can be effectively characterized by the combination of various values of degrees of freedom in all components. A small degrees of freedom in a component will contribute more to the heaviness of the tails of the overall conditional distribution. When all the component degrees of freedom tend to infinity, the TMAR model is reduced to the MAR model. Therefore, the MAR model is a limiting case of the TMAR model.

Kurtosis of Y_t, which is used to describe the degree of heavy-tailedness of the distribution, can be computed. The condition for the existence of the fourth-order moment of a stationary TMAR model can be derived. For illustrative purpose, we only consider the TMAR(g; 0, …, 0) model with $\phi_{k0} = 0$, for $k = 1, \ldots, g$. Note that the first-order and third-order moments of this model are both zeros while the second-order moment is $\sum_{k=1}^{g}\alpha_k\sigma_k^2$. The existence condition of the fourth-order moment is given in the following theorem.

Theorem 3. The fourth-order moment of a stationary TMAR (g; 0, …, 0) model with $\phi_{k0} = 0$, for $k = 1, \ldots, g$ exists if and only if $\nu_k > 4$.

The existence condition of the fourth-order moment for TMAR is the same as those for the Student *t* distribution. The expression of the fourth-order moment for a stationary TMAR (g; 0, …, 0) model with $\phi_{k0} = 0$, for $k = 1, \ldots, g$, is given by $3\sum_{k=1}^{g}\left\{\alpha_k\sigma_k^4\left(\nu_k-2\right)\right\}/\left(\nu_k-4\right)$ where $\nu_k > 4$. It can be shown that the kurtosis of Y_t is generally greater than 3 (i.e., the TMAR model caters for heavier conditional distributions as compared to Gaussian processes). It can be noted that the expression of the fourth-order moment for TMAR model is similar to that for MAR model with the additional terms $\left(\nu_k-2\right)/\left(\nu_k-4\right)$ while the expressions of the variance are the same for both models. Since the additional terms are larger than 1 unless $\nu_k \to \infty$, the (marginal) distribution from the TMAR model can be much more heavy-tailed than that from MAR model.

The autocorrelations for the TMAR model are similar to that for MAR models. They satisfy a system of equations similar to the Yule-Walker equations. For a second-order stationary process Y_t that follows an TMAR model, it is easy to show that $\rho_j=\sum_{i=1}^{p}\left(\sum_{k=1}^{g}\alpha_k\phi_{ki}\right)\rho_{|j-i|}$, for $j = 1, \ldots, p$ where ρ_j is the lag j autocorrelation. Note that these equations are similar to the Yule-Walker equations for the ordinary AR(p)

process where the coefficient $\sum_{k=1}^{g} \alpha_k \phi_{ki}$ replaces the lag i coefficient of the AR(p) process. The range of possible autocorrelations is as great as that of the standard AR process, since the AR model is just a limiting case of the TMAR model.

2.2. Estimation

The Student t density function can be obtained from a normal scale mixture model (see, e.g., Peel and McLachlan, 2000). Suppose that the observation X is generated from the following model, $\int \phi\left(x;\mu,\sigma^2/w\right) h\left(w\right) dw$, where $\phi(\cdot)$ denotes the p.d.f. of the normal distribution with mean μ and variance σ^2, and $h(\cdot)$ is a p.d.f. of any distribution. The function $h(\cdot)$ is employed to control the magnitude of the variance in the normal density function. Assume that we choose the gamma density function for $h(\cdot)$, then the random variable W is distributed as $W \sim \text{gamma}\left\{\nu/2,(\nu-2)/2\right\}$. After integrating out w, we get a standardized Student t distribution with location parameter μ, scale parameter σ and degrees of freedom ν.

The log-likelihood for the TMAR model is constructed by the normal scale mixture model. Note that the log-likelihood does not admit a closed form solution if the Student t density function is applied directly (McLachlan and Krishnan, 1997). Assume that we have observations $Y = (y_1, y_2, \ldots, y_n)$ generated from a TMAR model. Let $Z = (Z_1, \ldots, Z_n)$ be a $g \times n$ unobservable random matrix, where $Z_t = (Z_{kt})$, for $t = 1, \ldots, n$, is a g-dimensional column indicator vector showing the origin of the kth observation, i.e., $Z_{kt} = 1$ if y_t is generated from the kth component of the model and $Z_{kt} = 0$ otherwise. Analogous to the formulation of Z, we consider another missing random matrix, $W = (W_1, \ldots, W_n)$, where W_t is also a g-dimensional vector. Given $z_{kt} = 1$, the conditional distribution of W_t is $\left\{W_{kt} \middle| z_{kt} = 1\right\} \sim \text{gamma}\left\{\nu_k/2,(\nu_k-2)/2\right\}$ and $W_1, \ldots, W_n$ are distributed independently. Let $\alpha = \left(\alpha_1,\ldots,\alpha_{g-1}\right)'$, $\nu = \left(\nu_1,\ldots,\nu_g\right)'$, $\theta = \left(\theta_1',\ldots,\theta_g'\right)'$ with $\theta_k = \left(\phi_{k0},\phi_{k1},\ldots,\phi_{kp_k},\sigma_k\right)'$, $k = 1,\ldots,g$, and the vector of parameters of the TMAR model is defined as $\Psi = \left(\alpha',\theta',\nu'\right)'$. The conditional log-likelihood is $\ell = \ell_1(\alpha) + \ell_2(\nu) + \ell_3(\theta)$ where

$$\ell_1(\alpha) = \sum_{k=1}^{g} \sum_{t=p+1}^{n} Z_{kt} \log\left(\alpha_k\right),$$

$$\ell_2(\nu) = \sum_{k=1}^{g} \sum_{t=p+1}^{n} Z_{kt} \left[-\log\left\{\Gamma\left(\frac{1}{2}\nu_k\right)\right\} + \frac{1}{2}\nu_k \log\left(\frac{\nu_k - 2}{2}\right) + \frac{1}{2}\nu_k \left(\log W_{kt} - W_{kt}\right) - \log\left(W_{kt}\right) + W_{kt} \right],$$

$$\ell_3(\theta) = \sum_{k=1}^{g} \sum_{t=p+1}^{n} Z_{kt} \left[-\frac{1}{2}\left\{\log(2\pi) + \log\sigma_k^2 - \log W_{kt}\right\} - \frac{e_{kt}^2 W_{kt}}{2\sigma_k^2} \right].$$

Here, $e_{kt} = y_t - \phi_{k0} - \phi_{k1} y_{t-1} - \ldots - \phi_{kp_k} y_{t-p_k}$.

The EM algorithm (Dempster et al., 1977) is adopted for the estimation of the parameters of the TMAR model. The standard errors of the parameter estimates can be computed by Louis' method (1982), after the EM estimation. The details of the EM estimation algorithm for estimating the class of TMAR models are given in Wong et al. (2009).

2.3. Model selection

There are two aspects of model selection in the TMAR models, namely, the number of components (g) and the orders of each AR component (p_k). We propose to use the Bayesian information criterion (BIC) (Schwarz, 1978) for model selection. Although the use of BIC to choose g is somewhat non-standard as it corresponds to testing problems with nuisance parameters that do not exist under the null hypothesis (Davis, 1987), it can serve as a rough guide in model selection. Wong et al. (2009) illustrate the performance of the minimum BIC procedure with simulation studies. They find that the minimum BIC procedure performs well.

3. THE HONG KONG HANG SENG INDEX

The Hong Kong Hang Seng Index is a market-valued weighted index of constitute stocks that are representative of the Hong Kong market. Our study period starts from January 2, 1996 and ends on 30 December, 2005 (2471 daily observations). The data set is taken from the webpage of *Yahoo! Finance*, which is publicly available.

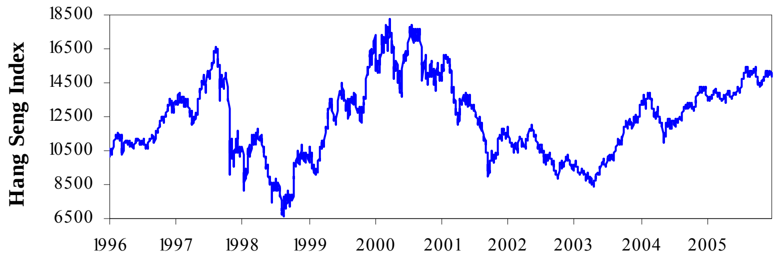

Figure 1. The Hong Kong Hang Seng Index.

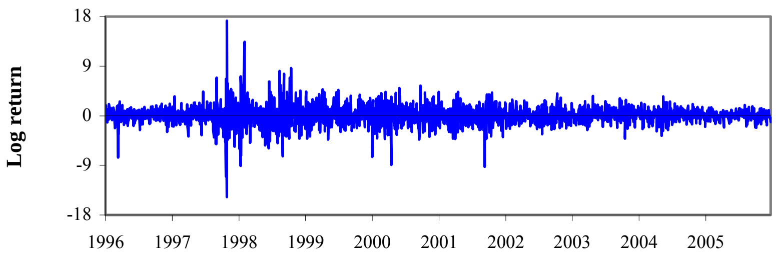

Figure 2. The log return series of the Hong Kong Hang Seng Index.

The original index series y_t and the log return series r_t (2470 observations) are shown in Figures 1 and 2 respectively. It is clear that the original series y_t is non-stationary. The log return series appears to be stationary. It can be noticed that Hong Kong market was greatly affected by the Asian Financial Crisis in October 1997. The Hang Seng Index dropped by 14.73% on 28 October 1997 and rose by 17.25% on 29 October 1997. The excess kurtosis of log return series is 11.09 which suggest that models allied with Gaussian assumption may not describe the data well.

We find that a TMAR(3;0,6,6) model is the best fitted model to the return series. It minimizes the BIC over all two- and three-component TMAR models. The parameter estimates and standard errors are shown in Table 1. Note that it is a TMAR model without intercept. In fact, we have considered models with intercepts. However, none of them is chosen by the minimum BIC procedure. The value of BIC is 8935.15. A close examination of this model reveals that the two AR(6) components are both non-stationary. However, after mixing two non-stationary components with a white noise component, the resulting TMAR model is stationary. This is a particularly nice feature of mixture-type time series models. Note that as all $\nu_k > 4$, the kurtosis exists for this TMAR model.

Table **1**. Estimation result of TMAR(3;0,6,6) model

k		ν_k	α_k	σ_k	ϕ_{k1}	ϕ_{k2}	ϕ_{k3}	ϕ_{k4}	ϕ_{k5}	ϕ_{k6}
1	Estimate	4.2330	0.5612	1.0633						
	s.e.	1.3521	0.0444	0.1375						
2	Estimate	4.7215	0.2489	1.7957	0.0656	-0.2714	-0.2196	-0.5494	-0.2866	-0.2460
	s.e.	1.2684	0.0336	0.1543	0.0840	0.0548	0.0537	0.0546	0.0498	0.0635
3	Estimate	4.2271	0.1899	1.3489	0.4100	0.0934	0.4959	0.6145	0.2524	0.3661
	s.e.	1.7638	0.0231	0.2100	0.0636	0.0540	0.0510	0.0660	0.0535	0.0565

For comparison, MAR model is also fitted to the return series. The best MAR model is MAR(3;0,6,6) model without intercept and the value of BIC is 8983.19. The parameter estimates and standard errors are shown in Table 2. Note that the fitted MAR model also comprises of two non-stationary AR(6) components. The structures of TMAR(3;0,6,6) and MAR(3;0,6,6) are actually similar except that in TMAR model, the degrees of freedom parameters, ν_k, in the *t* distributions are used to control the shape of conditional distributions.

Besides MAR model, we try to compare the TMAR model with autoregressive (AR), moving average (MA), random walk (RW), generalized autoregressive conditional heteroscedastic (GARCH) and GARCH with Student *t* distribution (GARCH-*t*) models on the basis of their ability to describe the predictive distribution of the return series. The fitted models are selected based on minimum BIC Criterion which are shown in Table 3. Note that the processes implied by both fitted GARCH-type models are close to non-stationary as the sums of the GARCH parameters are close to one.

Table **2**. Estimation result of MAR(3;0,6,6) model

k		α_k	σ_k	ϕ_{k1}	ϕ_{k2}	ϕ_{k3}	ϕ_{k4}	ϕ_{k5}	ϕ_{k6}
1	Estimate	0.6064	0.8852						
	s.e.	0.0324	0.0402						
2	Estimate	0.2168	2.0700	0.0240	-0.2543	-0.2393	-0.5440	-0.4450	-0.2647
	s.e.	0.0253	0.0948	0.0641	0.0565	0.0745	0.0636	0.0648	0.0723
3	Estimate	0.1768	1.4120	0.4598	0.2158	0.5586	0.5046	0.3408	0.3557
	s.e.	0.0198	0.0912	0.0596	0.0549	0.0806	0.1017	0.0622	0.0659

Table **3**. Estimation results of commonly used time series models

Model	Fitted Model
AR(4)	$r_t = 0.0330\, r_{t-1} - 0.0546\, r_{t-2} + 0.1006\, r_{t-3} - 0.0561\, r_{t-4} + \varepsilon_t$ where $\varepsilon_t \sim N(0, 2.8803)$
MA(3)	$r_t = \varepsilon_t + 0.0364\, \varepsilon_{t-1} - 0.0472\, \varepsilon_{t-2} + 0.0925\, \varepsilon_{t-3}$ where $\varepsilon_t \sim N(0, 2.8883)$
RW	$r_t = \varepsilon_t$ where $\varepsilon_t \sim N(0, 2.9246)$
AR(1)-GARCH(1,1)	$r_t = 0.0632\, r_{t-1} + \varepsilon_t,\ \varepsilon_t / \sqrt{h_t} \sim N(0,1)$ where $h_t = 0.0151 + 0.0709\, \varepsilon_{t-1}^2 + 0.9255\, h_{t-1}$
GARCH(1,1)-*t*	$r_t = \varepsilon_t,\ \varepsilon_t / \sqrt{h_t} \sim t_{5.7851}$ where $h_t = 0.009289 + 0.0511\, \varepsilon_{t-1}^2 + 0.9469\, h_{t-1}$

The empirical coverages of the in-sample one-step-ahead prediction intervals, based on $F\left(r_{t+1} \middle| \mathcal{F}_t\right)$, t = 9, 10, …, 2470, for the return series generated by each model are given in Table 4. If a model describes the conditional distributions of the data well, the empirical coverages of the one-step-ahead prediction intervals generated should be close to the nominal levels. Note that the absolute percentage deviations of the empirical coverage to the nominal level are shown in brackets. Generally speaking, performance of prediction under TMAR model is better than those of MAR, AR, MA, RW, AR-GARCH and GARCH-*t* models. The empirical coverages of the TMAR-based prediction intervals are closer to the nominal coverages. On the other hand, for all models, the deviations of empirical coverages from the nominal coverages are increased as the level of prediction intervals is decreased.

Table **4**. Empirical coverage of the (1 – α)100% prediction intervals for the Hong Kong Hang Seng Index

	(1 – α)100%							
Model	99	98	95	90	80	70	60	50
TMAR(3;0,6,6)	99.11	97.93	95.25	90.29	79.85	69.12	60.12	50.39
	(0.11)	(0.07)	(0.26)	(0.32)	(0.19)	(1.26)	(0.20)	(0.78)
MAR(3;0,6,6)	98.94	98.17	95.94	91.10	80.41	69.69	60.30	50.95
	(0.06)	(0.17)	(0.99)	(1.22)	(0.51)	(0.44)	(0.50)	(1.90)
AR(4)	97.77	96.63	94.76	92.20	85.86	79.40	71.27	63.02
	(1.24)	(1.40)	(0.25)	(2.44)	(7.33)	(13.43)	(18.78)	(26.04)
MA(3)	97.60	96.75	94.76	92.08	85.94	79.60	71.52	63.47
	(1.41)	(1.28)	(0.25)	(2.31)	(7.43)	(13.71)	(19.20)	(26.94)
RW	98.09	96.67	94.64	92.08	85.94	80.50	72.45	63.84
	(0.92)	(1.36)	(0.38)	(2.31)	(5.08)	(15.00)	(20.75)	(27.68)
AR(1)-GRACH(1,1)	98.21	97.20	94.80	89.80	81.63	73.43	65.79	57.05
	(0.80)	(0.82)	(0.21)	(0.22)	(2.04)	(4.90)	(9.65)	(14.10)
GARCH(1,1)-*t*	99.11	98.21	95.33	88.99	78.46	69.20	60.46	51.89
	(0.11)	(0.21)	(0.35)	(1.12)	(1.93)	(1.14)	(0.77)	(3.78)

We generated the empirical coverages of one-sided (lower) prediction intervals for the return series. Note that the lower prediction intervals for return series are related to the concept of Value-at-Risk in finance literature. The results are recorded in Table 5. The performance of lower prediction intervals under the TMAR model is better than those under the MAR, AR, MA, RW and AR-GARCH models. On the other hand, the performance of TMAR-based lower prediction intervals is similar to that of GARCH-*t*-based lower prediction intervals.

Table **5**. Empirical coverage of the $(1 - \alpha)100\%$ one sided (lower) prediction intervals for the Hong Kong Hang Seng Index

Model	$(1 - \alpha)100\%$		
	99	98	95
TMAR(3;0,6,6)	98.90	98.09	95.61
	(0.10)	(0.09)	(0.64)
MAR(3;0,6,6)	99.02	98.33	95.65
	(0.02)	(0.34)	(0.68)
AR(4)	98.29	97.60	95.94
	(0.72)	(0.41)	(0.99)
MA(3)	98.33	97.56	96.02
	(0.68)	(0.45)	(1.07)
RW	98.37	97.44	95.81
	(0.64)	(0.57)	(0.85)
AR(1)-GRACH(1,1)	98.54	97.68	94.88
	(0.46)	(0.33)	(0.13)
GARCH(1,1)-*t*	98.86	98.05	94.47
	(0.14)	(0.05)	(0.55)

4. DISCUSSION AND CONCLUSION

We have successfully applied the class of Student *t*-mixture autoregressive models to a stock index data. This class of processes has the advantage of being able to capture time-varying means, variances and the shape of conditional distributions, which are stylized facts in many observed financial and economic time series. (Granger and Teräsvirta, 1993; Franses and van Dijk, 2000; and Tsay, 2002). They can be employed to model a wide range of possible dynamics for the economic and finanical time series data.

When the number of component in a TMAR model equals to one (i.e., $q = 1$), the model reduces to a linear AR process. Therefore, it would be useful to derive a test for $q = 1$ in a TMAR process. However, such test is somewhat non-standard as it corresponds to testing problems with nuisance parameters that do not exist under the null hypothesis (Davis, 1987). It is also interesting to examine the possibility of extending the TMAR process with GARCH errors. Research in these directions is in progress.

ACKNOWLEDGMENTS

This work was supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region (Competitive Earmarked Research Grant Project No. CUHK4733/05H).

REFERENCES

Davies, R.B. (1987), Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika*, 74, 33–43.

Dempster, A.P., Laird, N.M., and Rubin, D.B. (1977), Maximum likelihood from incomplete data via the EM algorithm. *Journal of Royal Statistical Society*, Series B, 39, 1–38.

Franses, P. H., and van Dijk, D. (2000). *Nonlinear Time Series Models in Empirical Finance*. Cambridge: Cambridge University Press.

Granger, C.W.J., and Teräsvirta, Y. (1993). *Modelling Nonlinear Economic Relationships*. Oxford: Oxford University Press.

Louis, T.A. (1982), Finding the observed information matrix when using the EM algorithm. *Journal of Royal Statistical Society*, Series B, 44, 226–233.

McLachlan, G.J., and Krishnan, T. (1997). *The EM Algorithm and Extensions*. New York: Wiley.

Peel, D., and McLachlan, G.J. (2000), Robust Mixture Modelling using the *t*-Distribution. *Statistics and Computing*, 10, 339–348.

Schwarz, G. (1978), Estimating the Dimension of a Model. *The Annals of Statistics*, 6, 461–464.

Tong, H. (1990). *Non-linear Time Series*. New York: Oxford University Press.

Tsay, R.S. (2002). *Analysis of Financial Time Series*. New York: John Wiley & Sons.

Wong, C.S., Chan, W.S., and Kam, P.L. (2009), A Student *t*-mixture autoregressive model with applications to heavy-tailed financial data. To appear in *Biometrika*.

Wong, C.S., and Li, W.K. (2000), On a mixture autoregressive model. *Journal of the Royal Statistical Society*, Series B, 62, 95–115.