

## Conditional Beta Capital Asset Pricing Model (CAPM) and Duration Dependence Tests

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### Abstract

This paper uses a sample of 50 companies continuously listed on Main Board of Bursa Malaysia from January 1994 until December 2001 and uses duration dependence tests whilst applying two asset pricing models based on the CAPM; the two Factor Model developed by Fama and French (F&F)(1998) and Ferson, Sarkissian and Simin's (FSS) (2008) conditional beta model applied to estimate the conditional beta of CAPM as to generate the positive and negative abnormal returns. The findings suggest that both the Log Logistic and Weibull hazard models seem to support the existence of negative duration dependence for both positive and negative runs of abnormal returns, consistent with the presence of bubbles theory as predicted by McQueen and Thorley (1994). The negative runs of abnormal returns for both the F&F and FSS models show that more than 80% of the sample seems to support the existence of negative duration dependence using both hazard models. Meanwhile the positive runs show that not more than 80% of the sample rejects the null hypothesis based on LR tests of the absence of duration dependence. This study also compare whether the estimation of the run lengths of positive and negative abnormal returns for both F&F and FSS models are significantly different. The results suggest that the number of runs for both models by F&F and FSS are significantly different.

**Keywords:** *Duration dependence, Two Factor Models, rational bubbles, Log Logistic and Weibull Hazard Models*

## 1. Introduction

The issue of whether stock markets are rational is a central one. (In this paper, the concern is whether the Malaysian stock market is rational?). One way of approaching this issue is by means of the concept of rational bubbles. Bubbles can be defined as the phenomena of stock prices moving consistently away from fundamental values. A recent study conducted by Chan, McQueen and Thorley (1998) used the rational bubble concept to assess whether the US market as represented by the Standard and Poor 500 Index, is rational through a technique called a duration dependence test. However, they found that there is no duration dependence in this market. Harman and Zuehlke (2004) disagreed and asserted that there is significant duration dependence evident in runs of both positive and negative abnormal returns that are inconsistent with the model of rational bubbles as proposed by McQueen and Thorley (1994).

Watanapalachaikul and Islam (2007) also investigated the existence of rational bubbles in the Thai stock market. Similar to Chan et al. (1998), they employed the duration dependence test but they used two hazard models namely (i) the Log-Logistic and (ii) the Weibull Hazard Model. In addition, they also analysed rational bubbles using annual data. This is in contrast to the previous studies (McQueen and Thorley, 1994; Chan et al., 1998), which used one-short period of analysis. Based on their study, Watanapalachaikul and Islam (2007) contended that it was easier to detect the exact size of bubbles in each particular year using a yearly based analysis.

In another study conducted by Harman and Zuehlke (2004) further support was provided for Watanapalachaikul and Islam (2007). They employed the generalized Weibull hazard model on value weighted portfolios of NYSE stocks from 1927 through 1997. This model provides much more flexibility which allows changes in the direction of duration dependence. Their findings suggest that duration dependence is not monotonic. Basically, the idea is of duration elasticity being initially positive. However, the sign can be negative as the duration of the run of abnormal returns increases. The presence of significant duration dependence in runs of negative abnormal return returns is inconsistent with McQueen and Thorley (1994). McQueen and Thorley (1994) suggested that the duration dependence pattern is indicative of the existence of bubbles since it cannot be the result of asymmetric or leptokurtic innovations in fundamentals alone. Though a study by Harman and Zuehlke (2004) agreed with Mudholkar, Srivasta, and Kollia (1996) who reported negative duration over much of the range of data they examined, and it is evidence in support of both positive and negative runs of abnormal returns.

Another study conducted by Harman and Zuehlke (2004) applied the duration dependence test to both equally weighted and value weighed portfolios of NYSE traded securities and found that both portfolios produced evidence of negative and positive duration dependence, which once again was not supported by the model proposed by McQueen and Thorley (1994). One potential explanation for this is a failure of the assumption of market efficiency.

The initial research on the existence of rational bubbles in equity securities on the Malaysian stock market was conducted by Chan et al. (1998), using monthly and weekly Kuala Lumpur Composite Index returns from January 1977 through April 1994 employing duration dependence tests using a Log-Logistic Hazard Model. They found that there was no evidence of the existence of rational bubbles in the Malaysian stock market between the years 1977 until 1999. However, in this paper, the focus will be on a sample of Main Board companies from the Kuala Lumpur Stock Exchange (Bursa Malaysia).

The performance of the South East Asian economies has been relatively erratic throughout the 1990s and until 2007, most significantly during the 1997 Asian financial crisis. Do rational bubbles in the equity premium exist in the Malaysian stock market? The present study attempts to detect rational bubbles in the equity premium using Duration Dependence Tests. In these tests two models are employed: (1) the Log Logistic Hazard Model and (2) the Weibull Hazard Model.

There are a few relevant papers published about the Malaysian stock market, and in one of the latest papers, published by Mokhtar et al. (2006), it was revealed that bubbles did not exist during the economic crisis of 1997. However, the detection of rational bubbles in this study limits their analysis to a study of sectoral indices data rather than individual stocks, as opposed to the use of individual stocks that we adopt for

forecasting the equity premium. Furthermore, this is the first study of Malaysian data to employ a conditional beta version of CAPM. We apply two models which are; (1) the two Factor model developed by Fama and French (1998) and an asset pricing model with time varying coefficients as developed by Ferson, Sarkissian and Simin (2008) to estimate the abnormal returns and utilise them in duration dependence tests.

This paper contains 4 sections which include an introduction, a data and methodology section, an explanation of the estimation of the hazard model and a further section presenting the results followed by a concluding section.

## 2. Data and Methodology

We adopt the Duration dependence test using the Log Logistic Hazard Model and Weibull Hazard Model that are more widely accepted (Fung 2001; Harman and Zuehlke 2004), because of their robustness in testing for rational bubbles. Furthermore, both hazard models are chosen because of their flexibility in applying the hazard rate distribution. The paper uses monthly aggregate returns on the Malaysian Stock Exchange for the period 1994 to 2001 and collected the data for dividend price ratios and dividend yields from Professor Kenneth French’s personal website (data for Malaysia). These will be analysed using regressions applying the two factor model of Fama & French (1998) and a time varying coefficient model by Ferson, Sarkissian and Simin (2008). These models are used to assess the relationship between the equity premium and dividend yields. The dividend yield has been chosen as an independent variable as the initial regression result shows significantly ability to explain the equity premium. The model can be illustrated as follows:

### The two Factor Model By Fama and French (1998)

$$R_t - R_{ft} = \alpha + \beta_0 (R_m - R_f) + \beta_1 (HB - LB / M)_t + u_t \quad (1)$$

$$\beta^* = \beta_0 + \beta_1 (HB - LB / M)_t$$

Where, HB and LB are defined as high and low book value to market value respectively.

### Asset Pricing Time Varying coefficients model by Ferson, Sarkissian and Simin (FSS) (2008)

$$R_t = \alpha_0 + \alpha_1 Z_{t-1} + \beta_0 (R_{m,t}) + \beta_1 (R_{m,t} Z_{t-1}) + u_t \quad (2)$$

$$\beta^* = \beta_0 + \beta_1 (Z_{t-1})$$

Where the  $Z_{t-1}$  is the lagged predictor variable. For the purposes of this paper, one month lagged dividend yields are used to estimate conditional betas. The regression models are used to forecast aggregate returns (the equity premium) and to measure realised excess returns. These excess returns are then subjected to duration dependence tests based on hazard models and the Log Logistic and Weibull distributions.

The FF model and FSS model assume that the both the two factor and conditional CAPM imply that  $\alpha_0=0$  and  $\alpha_1=0$ . Where, if the model holds, then  $E(u_t) = E(u_t [R_{m,t} - R_{f,t}]) = 0$  and  $E(u_t) = E(u_t [R_{m,t} Z_{t-1}]) = 0$  respectively. Once the betas have been estimated, then equation (1 and 2) are used to calculate the forecast stock returns and can then be applied to identify abnormal returns for both positive and negative series for the duration dependence tests. Thus the sample period adopted for this paper is shown in table 1:

**Table 1: Period of Estimating coefficient and forecast for duration Dependence Test**

| Model  | Benchmark                          | Total Period                       | Estimation period                   | Forecast Period                     |
|--|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| Two Factor Model * (Model 1)<br>Fama and French (1998)   | 50 companies of<br>Main Board KLSE | January 1994 until<br>October 2001 | January 1994 until<br>November 1997 | December 1997 until<br>October 2001 |
| Conditional beta Based on Asset<br>pricing Time varying* (Model2)<br>Ferson, Sarkissian & Simin (2008) | 50 companies of<br>Main Board KLSE | January 1994 until<br>October 2001 | January 1994 until<br>November 1997 | December 1997 until<br>October 2001 |

Using the LR test, both versions of the CAPM models (F&F and FSS) are tested based on the following hypothesis:

- H<sub>0</sub>: There is no rational bubble. Excess returns do not exist.  
 There is no duration dependence; there is a constant hazard rate.  
 H<sub>1</sub>: There are rational bubbles. Excess returns do exist.  
 There is positive or negative duration dependence; there are decreasing or increasing hazard rates.

### 3. Log Logistic and Weibull Hazards Model

The Log Logistic Hazard Model is defined as estimated by McQueen and Thorley (1994) and Harman and Zuehlke (2004):

$$\ln L(\alpha, \beta) = \sum_{i=1}^N \{J_i \ln[g(t_i)] + (1 - J_i) \ln[1 - G(t_i)]\} \quad (3)$$

where  $\alpha$  is the shape parameter of the lognormal distribution,  $\beta$  is the duration elasticity of the hazard function,  $J_i$  is a duration of the process or time to exit from a state,  $g_i$  is the discrete function for duration, and  $G_i$  is the corresponding distribution function. The discrete density and distribution functions for duration are related as:

$$G(t_i) = \sum_{k=1}^{t_i} g(k) \quad (4)$$

However, if the law of conditional probability is applied (Harman and Zuehlke, 2004), the density for completed duration is:

$$g(k) = h(k) \prod_{m=0}^{k-1} [1 - h(m)] \quad (5)$$

For positive integer  $k$ , where  $h(0)$  is defined as zero.

In addition, McQueen and Thorley (1994) used the logistic distribution function  $\psi$  evaluated at a linear transformation of log duration as:

$$h(k) = \psi [\alpha + \beta \ln(k)] = \{1 + \exp[-\alpha - \beta \ln(k)]\}^{-1} \quad (6)$$

The Weibull hazard Model (Mudholkar, Srivastava and Kolia, 1996) is defined as

$$S(t) = \exp(-at^{b+1}) \quad (7)$$

where  $S(t)$  is the probability of survival in the data up to at least time (t), The corresponding hazard function is:

$$h(t) = \alpha (\beta + 1)t^\beta \quad (8)$$

where  $\alpha$  is the shape parameter of the Weibull distribution, and  $\beta$  is the duration elasticity of the hazard function. The fundamental assumption of the Weibull Hazard model is a linear relationship between the log of the hazard function and the log of duration, where:

$$\ln [h(t)] = \ln[\alpha (\beta + 1)] + \beta \ln(t) \quad (9)$$

Further information regarding Weibull Hazard Model can be found in Harman and Zuehlke (2004).

#### 4. Findings

The main objective of the paper is to illustrate whether the forecasting of returns using CAPM as in the Fama and French (1998) model or the conditional version of the Ferson, Sarkissian and Simin model (2008) as a filter, are appropriate in evaluating the existence of duration dependence in the resulting excess returns on the Malaysian Stock Market.

Based on the LR test for the absence of duration dependence, there is evidence of negative duration dependence in Malaysia stock market for the period for December 1997 until October 2001. Table 2 presents a summary of number of companies that rejected the null hypothesis of no duration dependence in excess returns.

**Table 2 Results summary: percentage of sample of 50 companies to detect the existence of duration dependence**

| Bubble Model                               | Log Logistic Hazard Model    |               |                                     |               | Weibull Hazard Model         |               |                                     |               |
|--|------------------------------|---------------|-------------------------------------|---------------|------------------------------|---------------|-------------------------------------|---------------|
| CAPM Model                                 | Fama and French Model (1997) |               | Ferson, Sarkissian and Simin (2006) |               | Fama and French Model (1997) |               | Ferson, Sarkissian and Simin (2006) |               |
| Abnormal Returns                           | Positive Runs                | Negative Runs | Positive Runs                       | Negative Runs | Positive Runs                | Negative Runs | Positive Runs                       | Negative Runs |
| Number of Companies reject null hypothesis | 35                           | 41            | 38                                  | 43            | 36                           | 42            | 40                                  | 45            |
| % of Sample reject null hypothesis         | <b>70</b>                    | 82            | <b>76</b>                           | 86            | <b>72</b>                    | 84            | <b>80</b>                           | 90            |

Notes:  $H_0$ = the null hypothesis for the absence of duration dependence

Table 2 shows that in the case of negative runs of abnormal returns for both the F&F and FSS model it seems more than 80% (82%, 86%, 84% and 90%) of the sample seem to support the existence of negative duration dependence using both hazard models. Meanwhile the positive runs suggest not more than 80% (70%, 76%, 72% and 80%) of the sample reject the null hypothesis at a 95% confidence level based on LR tests for the absence of duration dependence which indicates that there is both negative duration dependence and positive runs of abnormal returns for the period December 1997 until October 2001 (forecast period). Therefore, based on this analysis, there are positive and negative runs of abnormal return prevalent for both hazard models.<sup>1</sup>

**Table 3 Paired Sample t-test on the number of positive, negative and total runs of abnormal returns for both the F&F Model and FSS Model**

| Statistic          | Positive Runs F&F Model | Positive Runs FSS Model | Negative Runs F&F Model | Negative Runs FSS Model | Total Runs F&F Model | Total Runs FSS Model |
|--------------------|-------------------------|-------------------------|-------------------------|-------------------------|----------------------|----------------------|
| Mean               | 13.66                   | 12.42                   | 13.72                   | 12.66                   | 27.38                | 25.08                |
| Standard Deviation | 1.184                   | 2.081                   | 1.819                   | 2.309                   | 3.568                | 3.948                |
| Paired t- test     | 4.128<br>(0.0001)***    |                         | 3.409<br>(0.001)***     |                         | 4.195<br>(0.0001)*** |                      |
| Correlation        | 0.412<br>(0.003)***     |                         | 0.453<br>(0.001)***     |                         | 0.472<br>(0.001)***  |                      |

Notes: figures in the parentheses are the t-statistics and Correlation p-values. \* denotes significance at the 10% level, \*\* denotes significance at the 5% level and \*\*\* denotes significance at the 1% level.

The purpose of table 3 is to compare whether the estimation of the run lengths of positive and negative abnormal returns for both F&F and FSS models are significantly different. The results suggest that the number of runs for both models by F&F and FSS are significantly different. The paired sample t-tests suggest significant differences in the means of runs of positive and negative abnormal returns for all types of runs (Positive, negative and Total Runs). Lastly, the analysis of the hazard rate dictates that the Log Logistic results suggest that the series of abnormal return runs were decreasing while the Weibull Hazard rate found that the runs series were decreasing then increasing.

<sup>1</sup> Results for 50 companies Log Logistic and Weibull hazards model are available upon request.

## 5. Conclusion

The tests for duration dependence examine a particular nonlinear form of return predictability that is consistent with the presence of bubbles allowing predictability associated with time variation of the equity returns filtered through a pricing model. In this paper two factor model developed by Fama and French (1998) and the conditional model of Ferson, Sarkissian and Simin (2008) were used to generate return predictions that were then compared with actual returns to derive excess returns. These were then subjected to duration dependence tests. Both models suggest the existence of duration dependence in excess returns though tests on the means of the results of the two approaches suggest they are significantly different.

This paper featured novel tests of the implications and estimation of the run lengths of positive and negative abnormal returns using two models developed by F&F (1998) and FSS (2008). The overall results suggest that both positive and negative runs show negative duration dependence as the LR test for absence of duration dependence was rejected which is consistent with the evidence of McQueen and Thorley (1994).

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