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**Abstract:** A government's ability to forecast key economic fundamentals accurately can affect business confidence, consumer sentiment, and foreign direct investment, among others. Government forecasts are subject to error, as can be seen by the frequent revisions that are made to initial, and even revised, official forecasts. A government forecast based on an econometric model is replicable, whereas one that is not based on an econometric model is non-replicable. Governments typically provide non-replicable forecasts of economic fundamentals, such as the inflation rate and real GDP growth rate. In this paper, we develop a model to generate one or more non-replicable government forecasts, examine the measurement errors contained in non-replicable government forecasts, compare replicable and non-replicable government forecasts. An empirical example to forecast economic fundamentals for Taiwan shows the relevance of the proposed methodological approach. The empirical analysis shows that replicable and non-replicable government forecasts can be distinctly different from each other, that efficient and inefficient estimation methods, as well as consistent and inconsistent covariance matrix estimates, can lead to significantly different outcomes, that government forecasts of economic fundamentals can differ markedly between initial and revised forecasts, and that alternative models and methods can lead to differences in the accuracy of government forecasts.

**Keywords:** Government expertise, efficient estimation, generated regressors, replicable government forecasts, non-replicable government forecasts, initial forecasts, revised forecasts, primary forecasts.

#### 1. Introduction

Governments typically provide non-replicable forecasts of economic fundamentals, such as the inflation rate and real GDP growth rate. A forecast is an inference about an event that was not observed at the time of the inference. A government's ability to provide initial and updated forecasts of key economic fundamentals accurately can affect, for example, business confidence, consumer sentiment, and foreign direct investment. Econometric models are frequently used to provide base-level forecasts in economics and business (see, for example, Franses and Legerstee (2009)). Such model-based forecasts can be adjusted by governments for a variety of reasons (see, for example, Franses (2008)). A government forecast that is based on an econometric model is replicable, whereas a government forecast that is not based on an econometric model is non-replicable. Governments can, and do, provide both replicable and non-replicable forecasts.

Government forecasts are subject to error, as can be seen by the frequent revisions that are made to initial, and even revised, official forecasts. In this paper, we develop an econometric model to generate replicable government forecasts, compare replicable and non-replicable government forecasts using efficient estimation methods, and present a direct test of expertise that is contained in government forecasts. An empirical example to forecast economic fundamentals for Taiwan shows the relevance of the methodological approach proposed in the paper. The empirical analysis shows that replicable government forecasts can lead to markedly different results, that alternative estimation and inferential methods can lead to significantly different outcomes, and that initial and revised government forecasts of economic fundamentals can differ substantially, so that alternative models and methods can and do lead to distinct differences in the accuracy of government forecasts.

The plan of the remainder of the paper is a follows. Section 2 presents the econometric model specification, analyses replicable and non-replicable government forecasts, presents the measurement error problem in obtaining initial and revised government forecasts, considers optimal forecasts and efficient estimation methods, and presents a direct test of expertise contained in government forecasts. The data analysis and a relevant empirical example are discussed in Section 3. Some concluding comments are given in Section 4.

### 2. Model Specification

In this section we present an econometric model to obtain initial and revised government forecasts. This will enable the generation of replicable government forecasts from non-replicable government forecasts, and permit a comparison to be made with non-replicable government forecasts.

Let the econometric model of the government for initial and revised forecasts for the variable of interest, y, be given as  $y = 2r + \beta_1 X_1^* + u_1, \quad u_1 \sim (0, \sigma_{11}^2 I),$  (1) where t = 1, ..., m, y is a  $T \times 1$  vector of observations to be explained (typically, an economic fundamental, such as

the inflation rate or the rate of growth of real GDP), Z is a  $T \times g$  matrix of T observations on g variables that are publicly available, and  $X_i^*$  is the latent expertise of government forecast i. It is also assumed that  $E(Z u_i) = 0$  and  $E(X_i^* u_i) = 0$ . The assumptions on the error term in (1) can be relaxed easily. If  $X_i^*$  were to comprise observable data, OLS for (1) would be consistent and efficient, and hence optimal in estimation. Under the assumption of correct specification and a mean squared error (MSE) loss function, the optimal forecast of y, given the information set, is its conditional expectation (see Patton and Timmermann (2007a, 2007b)). Let the  $T \times 1$  vector,  $X_i$ , represent the observable (that is, announced) government forecast i. The relationship between the non-replicable government forecast,  $X_i$ , and the expertise contained in government forecast i, is assumed to be given by

$$X_t = X_t^* + \eta_t, \eta_t \sim (0, \sigma_t^2 I)$$
<sup>(2)</sup>

where  $t = 1, ..., m, X_i, X_i^*$  and  $\eta_i$  are a  $T \times 1$  vectors, and  $\eta_i$  in (2) denotes the measurement error in government forecast *i*. It is assumed that  $X_i^*$  and  $\eta_i$  are uncorrelated for all *i*.

The observed non-replicable government forecast i is assumed to be modelled as

$$\mathcal{L}_{t} = \mathcal{W}_{t} \delta_{t} + \eta_{t}, \eta_{t} \sim (0, \sigma_{t}^{2} I)$$
(3)

where the  $T \times k_i$  matrix  $W_i$  is the information set available in obtaining the non-replicable government forecast *i* at time t-1. It is assumed that  $E(W_i \eta_i) = 0$  for all *i*,  $\delta_i$  is a  $k_i \times 1$  vector of unknown parameters, and

$$W_t \subset I_{-1}^t, \tag{4}$$

 $t = 1, ..., m, I_{-1}^{i}$  is the information set for the non-replicable government forecast *i* at time t-1. As Z is common knowledge, it follows from (4) that  $\{Z, W_i\} \subset I_{-1}^{t}$ , for all t = 1, ..., m. The information set  $I_{-1}^{i}$  is used to obtain optimal forecasts of *y* under a MSE loss function. It should be emphasized that an econometric model enables optimal forecasts to be generated, and hence the absence of an econometric model mean that optimal forecasts under a MSE loss function can not be obtained.

It follows from (3) and  $I_{-1}^{t}$  that

$$E\left(X_t | t_{-1}^t\right) \equiv X_t^* = W_t \hat{a}_t, \tag{5}$$

where  $W_t$  denotes the observable expertise of the non-replicable government forecast *i*. The rational expectation,  $E(X_t | L_1) - W_t \mathcal{O}_t$ , is a replicable government forecast, and its estimate is given as

$$\hat{X}_{t}^{*} - \hat{X}_{t} - W_{t}\hat{\delta}_{t} - W_{t}(W_{t}^{t}W_{t})^{-1}W_{t}^{t}X_{t} - P_{t}X_{t},$$
(6)

where  $P_t = W_t (W_t^* W_t)^{-1} W_t^*$  is the standard 'hat' matrix. Equation (6) shows that the latent government expertise for forecast *i*,  $X_i^*$ , can be obtained as an estimate of the observable non-replicable government forecast,  $\hat{X}_i$ . It is well known that the use of rational expectations reduces the number of unknowns in (5) from *T* to  $k_i$ , where  $k_i \ll T$  for all *i*.

Replacing the unobservable  $X_t^*$  in (1) with the observable  $\hat{X}_t$  gives

$$y = Zr + \beta_t \hat{X}_t + s_t \tag{7}$$

$$S_t = u_t - \beta_t P_t \eta_t \tag{8}$$

which is a composite error term, involving the measurement error,  $\eta_i$ , of non-replicable government forecast *i*. If  $\beta_t = 0$  for all *i*, in which case the government possesses no expertise but has access only to public information, it follows that  $\varepsilon_t = u_t$  for all *i*. The correlation between  $\bar{X}_i$  and  $\varepsilon_i$  is  $-\beta_t \delta_t^2 (T - k_t)$ , but OLS for the parameters in (7) is consistent as  $\hat{X}_i$  is asymptotically uncorrelated with  $\varepsilon_i$  for all *i*.

If  $u_i$  and  $\eta_l$  are mutually uncorrelated, then

$$V = E(s_t s_t') = E(u_t u_t') + \beta_t^2 P_t E(\eta_t \eta_t') P_t$$
  

$$V = \sigma_u^2 I + \beta_t^2 \sigma_t^2 P_t, \quad t = 1, \dots, m.$$
(9)

It is obvious that serial correlation and heteroskedasticity are present in (9) through the measurement error,  $\eta_i$ , in  $X_i$  in (2). Thus, if OLS is used to estimate (9), the correct covariance matrix in (9), or a consistent estimator such as the Newey-West HAC covariance matrix, should be used.

The necessary and sufficient conditions for OLS to be efficient in the presence of serial correlation and heteroskedasticity are given in Kruskal's theorem, of which a special case is the Gauss-Markov Theorem (see, for example, McAleer (1992), Fiebig et al. (1992), McAleer and McKenzie (1991), and more recently, Franses et al. (2009)), and are given by

- (i)  $VZ = ZA_1$ , for some  $A_1$ ;
- (ii)  $V\hat{X}_t = \hat{X}_t A_2$ , for some  $A_2$ .

Condition (i) is satisfied if  $\mathbb{Z} \perp W_t$  or if  $\mathbb{Z} \subset W_t$ , while condition (ii) is satisfied automatically as  $X_t = \mathbb{P}_t X_t$  in (6). In short, GLS is equivalent to OLS because the first step of the two step OLS estimator is satisfied as the transformation matrix is proportional to the data matrix.

Defining 
$$G_t = [Z_t : \hat{X}_t]$$
 and  $\phi_t^t = (\gamma_t^t, \beta_t^t)$  for all *i*, (7) may be rewritten as  
 $y = G_t \phi_t + \varepsilon_t.$  (10)

If conditions (i) and (ii) are satisfied, OLS is efficient for  $\phi_t$  and the correct OLS covariance matrix is given by

$$Var(\phi_t) = (G_t^{\prime}G_t)^{-1}G_t^{\prime}VG_t(G_t^{\prime}G_t)^{-1}, \qquad (11)$$
  
where V is given in (9). Substitution for V in (11) gives

$$Var(\hat{\phi}_{t}) = \sigma_{u}^{2} (G_{t}^{\prime}G_{t})^{-1} + \beta_{t}^{2} \sigma_{t}^{2} (G_{t}^{\prime}G_{t})^{-1} G_{t}^{\prime} P_{t} G_{t} (G_{t}^{\prime}G_{t})^{-1}, \quad (12)$$

which shows that the standard OLS covariance matrix of  $\phi_{i}$ , namely  $\sigma_{i}^{2}(G'_{i}G_{i})^{-1}$ , gives a downward bias in the covariance matrix and an upward bias in the corresponding t-ratios (see Pagan (1984) and Oxley and McAleer (1993) for examples in the case of generated regressors).

An alternative to estimating equation (7) is to substitute from (2) directly into (1) to obtain

$$y = Z\gamma + \beta_t (X_t - \eta_t) + u_t$$
  

$$y = Z\gamma + \beta_t X_t + (u_t - \beta_t \eta_t).$$
(13)

It is clear that OLS is inconsistent for (13) as  $X_i$  is correlated with  $\eta_i$ . Therefore, GMM should be used if the non-replicable government forecast,  $X_i$ , is used to explain the variable of interest, y. The effect of measurable government expertise,  $W_i$  on the non-replicable government forecast,  $X_i$ , can be tested directly in (3):

$$X_t = W_t \delta_t + \eta_t, \eta_t \sim (0, \sigma_t^2 I),$$

in which OLS is efficient given the information set. Moreover, the conditional expectation of  $X_i$  is an optimal forecast under a MSE loss function.

#### **3. Data and Empirical Analysis**

Since 1978, actual data and initial, primary and revised forecasts of economic fundamentals in Taiwan have been released by the government, as follows:

(i) in Q1 (February), release (initial) forecasts for Q1, Q2, Q3 and Q4 in the same year; and Q3 (primary value) and Q4 (revised forecast) in the previous year;

(ii) in Q2 (May), release (initial) forecasts for Q2, Q3 and Q4 in the same year; Q1 and Q2 for the following year; Q4 (primary value) for the previous year; and Q1 (revised forecast) in the same year;

(iii) in Q3 (August), release (initial) forecasts for Q3 and Q4 in the same year; Q1 (primary value) and Q2 (revised forecast) in the same year;

(iv) in Q4 (November), release (initial) forecasts for Q4 in the same year; Q1, Q2, Q3 and Q4 in the following year; and Q2 (primary value) and Q3 (revised forecast) in the same year.

Thus, there are several forecasts for each period, even considering just the one-quarter ahead forecasts, namely the initial forecast made in the same period, the primary forecast that is made available one quarter later, and the revised value that is available two quarters later. Only the initial forecast is a one-quarter forecast, with both the primary and revised forecasts being revisions of the initial forecast.

The data are obtained from the Quarterly National Economic Trends, Directorate-General of Budget, Accounting and Statistics, Executive Yuan, Taiwan, 1978-2008. The sample period used for the actual and government forecasts of seasonally unadjusted quarterly inflation rate and real growth rate of GDP is 1978 Q1 to 2008 Q1. Actual data on the inflation rate and real growth rate, as well as the initial and primary forecasts, are used in the empirical analysis. As there are some missing observations in the revised forecasts of both the inflation rate and real growth rate, revised forecasts are not considered in the empirical analysis.

The actual data, initial forecasts and primary forecasts of the inflation rate and real growth rate are reasonably similar, with most turning points being forecast accurately. The similarity in forecast performance is also shown in Table 1, which reports RMSE and MAD for the initial and primary forecasts of the inflation rate and real growth rate. Overall, it would seem that inflation rates are more accurately forecast than real growth rates. Moreover, it is not surprising to see that the primary forecast, which is an update of the initial forecast, provides a more accurate forecast of the two economic fundamentals using both forecast criteria.

Table 2 provides a formal test of the effects of government expertise on non-replicable initial and primary forecasts in equation (3). Government expertise for the primary forecast in (3) is approximated by one-period lagged real growth, one-period lagged inflation, one period lagged initial forecast, and one period lagged primary forecast, while government expertise for the initial forecast replaces the one period lagged primary forecast with its two period lagged counterpart. The lagged inflation rate is significant in both the non-replicable initial and primary forecasts of the inflation rate, and the lagged real growth rate is significant in both the non-replicable initial and primary forecasts of the real growth rate. Overall, the number of individually significant variables is greater for the non-replicable primary forecast counterparts.

The effects of the replicable initial and primary forecasts on the inflation rate and real growth rate in equation (7) are reported in Table 3, using OLS and both the OLS and Newey-West HAC standard errors. For the inflation rate, the replicable initial and primary forecasts are both highly significant, with the estimated coefficients being virtually

indistinguishable from unity, especially for the replicable primary forecast. A similar qualitative interpretation holds for the replicable initial and primary forecasts of the real growth rate, although the estimated coefficients are significantly greater than unity for both the replicable initial and primary forecasts. The biased OLS standard errors are considerably smaller than their Newey-West HAC counterparts, especially for the inflation rate. The goodness-of-fit of the replicable initial and primary forecasts use similar information sets.

Table 4 provides a formal test of the effects of the non-replicable initial and primary forecasts in equation (13) using OLS and GMM estimation. The instrument list for GMM for the primary forecast includes one-period lagged real growth, one-period lagged inflation, one-period lagged initial forecast, and one-period lagged primary forecast, while the instrument set for the initial forecast replaces the one period lagged primary forecast with its two period lagged counterpart. The OLS and GMM estimates are qualitatively the same in all cases, and are numerically quite similar for the non-replicable initial and primary forecasts for the inflation rate, and the non-replicable primary forecast of the real growth rate. The results in Table 4 suggest that the estimated coefficients of the non-replicable initial and primary forecasts of the inflation rate are indistinguishable from unity, as in Table 3, whereas those of the real growth rate are significantly greater than unity. However, the non-replicable primary forecasts of both the inflation rate and real growth rate would seem to be more accurate than their non-replicable initial forecast counterparts. Unlike the results in Table 3, the goodness-of-fit of the non-replicable initial and primary forecasts are not very close as the non-replicable forecasts do not use similar information sets.

In summary, the empirical results suggest that both the initial and primary forecasts are reasonably accurate measures of the inflation rate and the real growth rate for Taiwan. As the primary forecast is an updated measure of the initial forecast, it is not altogether surprising that it provides a more accurate forecast of both economic fundamentals.

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	Inflation		Real Growth Rate	
Forecasts	RMSE	MAD	RMSE	MAD
Initial	0.95	0.69	1.59	1.19
Primary	0.21	0.14	0.92	0.72

## **Table 1. Comparisons of Initial and Primary Forecasts**

Notes: RMSE and MAD denote root mean square error and mean absolute deviation, respectively. The sample period is 1978 Q1 to 2008 Q1. Data source: Quarterly National Economic Trends, Directorate-General of Budget, Accounting and Statistics, Executive Yuan, Taiwan, 1978-2008.

Included	Inflation		Real Growth Rate	
Variables	Non-Replicable	Non-Replicable	Non-Replicable	Non-Replicable
	Initial Forecast	Primary Forecast	Initial Forecast	Primary Forecast
Intercept	0.112	-0.351	1.285**	1.657**
	(0.283)	(0.331)	(0.283)	(0.358)
Real Growth (t-1)	0.056	0.084*	0.589**	0.584**
	(0.035)	(0.041)	(0.081)	(0.229)
Inflation (t-1)	0.865**	0.901**	0.012	0.005
	(0.125)	(0.302)	(0.024)	(0.030)
Initial Forecast (t-1)	0.018	0.030	0.068	-0.550**
	(0.158)	(0.189)	(0.136)	(0.155)
Primary Forecast (t-1)		0.006 (0.370)		0.622** (0.300)
Primary Forecast (t-2)	0.019 (0.084)		0.050 (0.081)	
Adjusted R <sup>2</sup>	0.916	0.896	0.787	0.740
F test	321.16**	254.69**	110.08**	84.96**

# Table 2. Testing the Effects of Expertise on Non-Replicable Initial and Primary Forecasts (standard errors in parentheses)

Notes: Expertise in (3) for the primary forecast is approximated by one-period lagged real growth, one-period lagged inflation, one period lagged initial forecast, and one period lagged primary forecast. Expertise for the initial forecast replaces the one period lagged primary forecast with its two period lagged counterpart. The F test is a test of expertise.

\* and \*\* denote significance at the 5% and 1% levels, respectively.

Estimation	Inflation				
Method	Intercept	Replicable Initial Forecast	Replicable Primary Forecast	Adjusted R <sup>2</sup>	
OLS	-0.347	1.040**		0.884	
	(0.188)	(0.035)			
HAC	[0.176]	[0.090]			
OLS	-0.042		1.001**	0.885	
	(0.180)		(0.033)		
HAC	[0.155]		[0.084]		
Estimation Method	Real Growth Rate				
	Intercept	Replicable	Replicable	Adjusted R <sup>2</sup>	
		Initial Forecast	Primary Forecast		
OLS	-0.662	1.223**		0.681	
	(0.495)	(0.077)		0.001	
HAC	[0.619]	[0.096]			
OLS	-2.694**		1.540**	0.665	
	(0.642)		(0.101)	0.005	
HAC	[0.788]		[0.143]		

# Table 3. Testing the Effects of Replicable Initial and Primary Forecasts (standard errors in parentheses)

Notes: Newey-West HAC standard errors are given in brackets. \*\* denotes significance at the 1% level.

Detimation	Inflation				
Method	Intercept	Non-replicable Initial Forecast	Non-replicable Primary Forecast	Adjusted R <sup>2</sup>	
OLS	-0.336**	1.035**		0.058	
	(0.110)	(0.020)		0.938	
GMM	-0.463**	1.098**		0.955	
Olvilvi	(0.095)	(0.027)		0.935	
	-0.048		1.003**	0.000	
OLS	(0.051)		(0.009)	0.990	
CMM	-0.034		1.018**	0.000	
Giviivi	(0.035)		(0.012)	0.990	
	Real Growth Rate				
Estimation	Tatanaa	Non-replicable	Non-replicable	A divisted $\mathbf{D}^2$	
Method	Intercept	Initial Forecast	Primary Forecast	Adjusted R	
OI S	-0.484	1.195**		0.830	
OLS	(0.317)	(0.048)		0.059	
GMM	-1.487**	1.329**		0.819	
	(0.481)	(0.070)		0.017	
	-0.127		1 119**		
OLS	(0.128)		(0.019)	0.968	
GMM	-0.150		1.122**	0.967	

### Table 4. Testing the Effects of Non-Replicable Initial and Primary Forecasts (standard errors in parentheses)

Notes: The instrument list for GMM for the primary forecast includes one-period lagged real growth, one-period lagged inflation, one-period lagged initial forecast, and one-period lagged primary forecast. The instrument set for the initial forecast replaces the one period lagged primary forecast with its two period lagged counterpart.

<sup>\*</sup> denotes significance at the 1% level.