A comparative study of groundwater evapotranspiration functions

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Abstract: Groundwater evapotranspiration (ET) is an important variable in surface-groundwater interaction modelling. In arid and semi-arid regions, an alarming amount of groundwater is evapotranspirated by crops in irrigated areas or phreatophytes along river valleys due to shallow groundwater tables. Groundwater evapotranspiration does not only reduce available water resources and thus water use efficiency, but also cause soil salt accumulation.

Even there is considerable debate, most groundwater modelling software packages (e.g. MODFLOW) use a simple linear function to describe the relation between depth to groundwater table and groundwater ET rate. It is also believed that the relation is non-linear and several non-linear functions have been proposed. We compared four widely used groundwater ET functions (linear, linear segment, power and exponential) in this paper. Lysimeter experiments were conducted from 1985 to 1991 at Huibei Irrigation Experiment Station, China, which is located in Liuyuankou Irrigation System (LIS) with a lot of water losses due to groundwater ET. Pan evaporation and groundwater ET rate data for different groundwater depths were collected and used to determine parameters of the ET functions by least squares method. Coefficient of determination and relative error were employed to measure how well the functions fit the observed data.

Values of coefficient of determination between measured and calculated values of groundwater ET rates for all the four functions were found to be high, which indicates that for the study site, they can effectively describe the relation between groundwater ET rate and groundwater depth if proper parameter values are chosen. Linear function is simple and it is suitable for shallow groundwater tables; while the power function has an exponent greater than 1 (1.77) which suggests non-linearity. Linear segment function requires more parameters while it can effectively simulate ET from deep groundwater. For simulation of regional groundwater ET in LIS, crop and soil types, parameter sensitivity and ease of optimization should be taken into consideration.

Keywords: *groundwater evapotranspiration, surface-groundwater interaction, groundwater modeling*

1. INTRODUCTION

In arid and semi-arid regions, an alarming amount of groundwater is evapotranspirated by crops in irrigated areas or phreatophytes along river valleys due to shallow groundwater depth. Groundwater evapotranspiration (ET) does not only reduce the available water resources and thus water use efficiency, but also cause soil salt accumulation (Khan et al, 2006; Szilagyi et al, 2004); therefore groundwater ET is usually an important component of regional water balance and simulation of groundwater ET has received increasing concerns.

In modeling the interactions between surface water and groundwater, ET functions are commonly used to estimate groundwater ET rates (e.g. McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996; Harbaugh et al, 2000; Banta, 2000; Harbaugh, 2005; Baird and Maddock, 2005; Li et al, 2008). ET function describes the relation between groundwater ET rate and groundwater depth.

Many groundwater ET functions, such as linear (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996; Harbaugh et al, 2000; Harbaugh, 2005), linear segment (Banta, 2000), power (Galperin et al, 1993), exponential (Shah et al, 2007), have been developed in the literature. Linear function is simple and requires only two parameters, ET surface and extinction depth. As noticed by Bauer et al (2004), even though there is considerable debate on the topic; most groundwater modeling software packages (e.g. MODFLOW-88 MODFLOW 96, MODFLOW 2000 and MODFLOW 2005) use the linear function. To better simulate the relation between evapotranspiration and groundwater depth, Banta (2000) proposed a new Evapotranspiration Package (ETS1) for MODFLOW. In ETS1, the relation of evapotranspiration rate to groundwater depth is conceptualized as a segmented line between ET surface and extinction depth, and the user can supply input to define as many intermediate segment endpoints as desired to define the relation of evapotranspiration rate to head between ET surface and extinction depth. The ETS1 package provides the capacity to accurately simulate groundwater ET; in using this package, however, it requires a set of parameters to describe the relation. A single nonlinear function may better describe the relation than the linear function and require fewer parameters than linear segment function. The power ET function is proposed by Aver'yanov (Galperin et al, 1993); it is also known as Kovda Function and is widely used in China. Recent work indicates that an exponential function better describes the change in groundwater ET with depth (Shah et al, 2007) and has been used by Li et al (2008).

Different conclusion may be drawn when using different ET functions to estimate groundwater ET. The objectives of this work include a) to compare the effectiveness and applicability of the above four functions in simulating groundwater evapotranspiration, b) to provide implication for selecting a suitable ET function for estimating regional groundwater ET.

2. MATERIALS AND METHODS

2.1. ET functions

Four widely used ET functions are compared in this work. Linear ET function is expressed by:

$$ET = \begin{cases} ET_{\max} & d < d_0 \\ ET_{\max} \cdot \dfrac{d_2 - d}{d_2 - d_1} & d_0 \le d \le d_2 \\ 0 & d > d_2 \end{cases} \quad (1)$$

Where ET is the groundwater ET rate (LT^{-1}); $ET_{\max}$ is the maximum ET rate (LT^{-1}); d is the depth of groundwater table (L); d_0 is the depth of ET surface (L); and d_2 is the cut-off or extinction depth (L) (Figure 1(a)).

In the ETS1 package of MODFLOW 2000 (Banta, 2000), it is assumed that the relation between depth to groundwater and evapotranspiration rate is non-linear. This package allows splitting up the relation between depth to groundwater and evaporation rate into a series of linear segments. The linear segment function is written as:

$$ET = \begin{cases} ET_{\max} & d < d_0 \\ ET_{\max,i} - \dfrac{ET_{\max,i+1} - ET_{\max,i}}{d_{i+1} - d_i}(d - d_i) & d_0 \le d_{i+1} < d \le d_i \le d_m \\ 0 & d > d_m \end{cases} \quad (2)$$

Where ET_i is the ET rate, where the groundwater level lies in the range of the i th segment (LT^{-1}); $ET_{\max,i}$ is the maximum ET rate of the i th segment (LT^{-1}); and h_i is the i th groundwater depth, where the ET rate reaches the maximum of the i th segment (L) ; m is the number of segments (Figure 1(b)).

Power function is also often used to depict the relation between ET rate and groundwater level. The power ET function is proposed by Aver'yanov (Galperin et al, 1993). In the original function, the ET surface concept is not adopted and here it is re-written as:

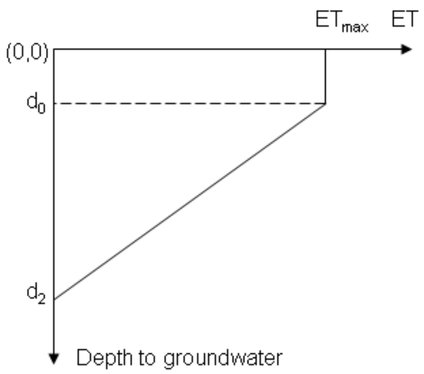
$$ET = \begin{cases} ET_{\max} & d < d_0 \\ ET_{\max} \cdot \left(\dfrac{d_2 - d}{d_2 - d_0} \right)^n & d_0 \le d \le d_2 \\ 0 & d > d_2 \end{cases} \quad (3)$$

Where n is an empirical coefficient depending on geology (-) (Figure 1(c)).

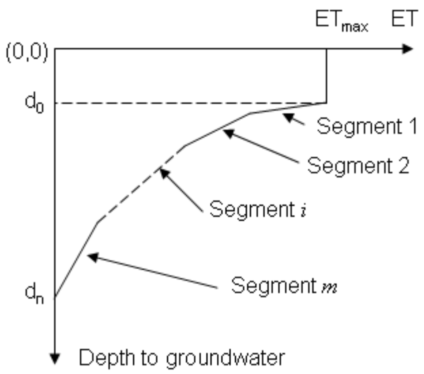
Exponential function is expressed as:

$$ET = \begin{cases} ET_{\max} & d < d_0 \\ ET_{\max} \cdot \exp\left(-a \cdot \dfrac{d - d_0}{d_2 - d_0} \right) & d_0 \le d \le d_2 \\ 0 & d > d_2 \end{cases} \quad (4)$$

Where a is also an empirical coefficient depending on geology (-) (Figure 1(d)).



(a)



(b)

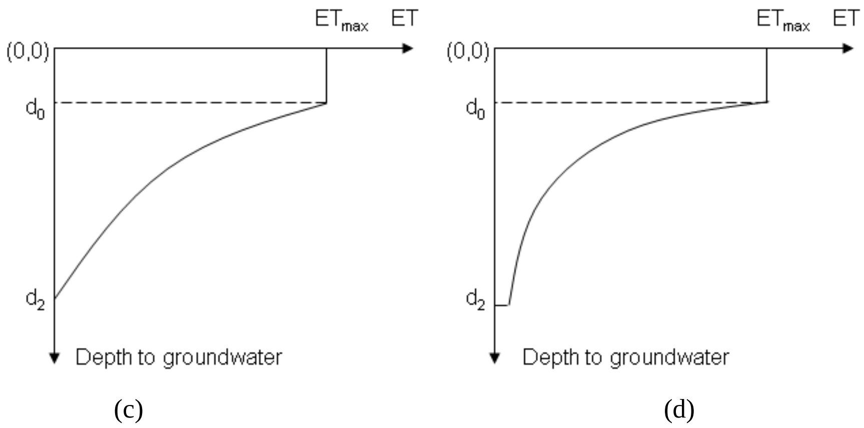


Figure 1. Sketches of groundwater ET functions: (a) linear, (b) linear segment, (c) power and (d) exponential

2.2. Experiments and data

Lysimeter experiments were conducted from 1985 to 1991 at Huibei Irrigation Experiment Station, Kaifeng, China. Huibei Irrigation Experiment Station (34^0 46'34''N, 114^0 30'35''E; 68.05 m altitude) is located in the middle of Liuyuankou Irrigation System (LIS) along the lower reach of Yellow River. Khan et al (2006) report that a significant amount of around 192 million cubic meters (MCM) of the irrigation water leaves LIS through fallow evapotranpiration and its reduction is therefore critical to increase water productivity, and believe that the greatest unaccounted flows from the LIS are through fallow evapotranpiration. Groundwater ET may be a considerable part of this fallow evapotranspiration due to irrigation-induced shallow groundwater tables. Relatively accurate estimates of groundwater ET from LIS are still not available for evaluating water use efficiency and soil salinisation risk. One of the objectives of this work is to determine the ET function for estimating regional groundwater ET in LIS.

The experiments comprised of groundwater depth, soil and crop treatments. The emphasis in this work was on various groundwater depths. There were 8 treatments, 0.45 m, 0.95 m, 1.45m, 1.95 m, 2.45 m, 2.95 m, 3.95 m and 4.95 m, among which 1.45 m and 2.45 with two repetitions (1988-1991) and other depth four repetitions (1985-1991). The 28 lysimeters were weighted on the 1st, 11th and 21st of each month and water balance components were measured and calculated for each ten-day (Zhu et al, 2002). Totally 1800 pairs of pan evaporation and ET rate were used to determine parameters of the ET functions.

2.3. Least-squares fitting and statistical comparison

The curves describing the relation between groundwater depth and evapotranspiration rate were fitted using the method of least squares. Least-squares fitting assumes that the best-fit curve of a given type is the curve that has the minimal sum of the deviations squared (least square error) from a given set of data. This approach has been widely used because it is simple and it gives good results for many cases (Wolberg, 2006).

Beside scatter diagrams, two statistical parameters, coefficient of determination (R^2) and relative error (RE) were used to measure how well the functions fit the observed data.

3. RESULTS AND DISCUSSION

3.1. Curve fitting

For all the four functions, depth of ET surface (d_0) and extinction depth (d_2) are common parameters. n in power function and a in exponential function also need to be determined. As for linear segment function, the depth between ET surface and extinction depth are divided into two segments. The maximum ET rate at depth of d_1 is set to $\alpha ET_{\max}$ ($0 < \alpha < 1$), then the functions for the two linear segments can be written as:

$$ET_1 = \frac{(1-\alpha)d - \alpha d_0 - d_1}{d_0 - d_1} \cdot ET_{\max} \quad \text{for} \quad d_0 < d < d_1 \tag{5}$$

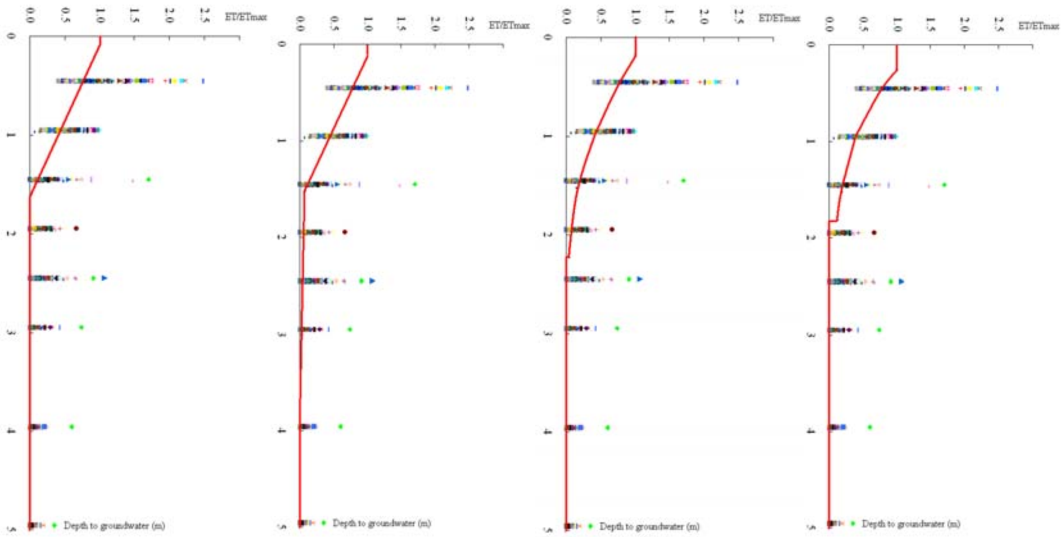
$$ET_2 = \frac{a(d-d_2)}{d_1-d_2} \cdot ET_{\max} \qquad \text{for } d_1 < d < d_2 \tag{6}$$

Where d_1 is a depth between d_0 and d_2.

Using the method of least squares, the relation is fitted by the four functions. Fitting results are shown in Table 1 and Figure 2. From Table 1, the depth of ET surface for linear and linear segment functions are both 0.1 m. For power and exponential functions, the values of d_0 are similar, 0.17 m and 0.26 m respectively. The order of ET surface depth is linear=linear segment<power<exponential. The linear function yields the lowest extinction depth (1.63 m) and the linear segment function can capture deepest groundwater ET (3.67 m), and the order of extinction depth is linear<exponential<power<linear segment. To eliminate the effect of different $ET_{\max}$, the fitted functions are plotted as $ET/ET_{\max} = f(d)$ (Figure 2). From Figure 2, even though the values of $ET/ET_{\max}$ for a certain depth vary significantly, all the curves can capture the feature of the relation between $ET/ET_{\max}$ and groundwater depth.

Table 1. Fitting results for the four functions

	d_0 (m)	d_1 (m)	d_2 (m)	α	n	a
Linear	0.10		1.63			
Linear segment	0.10	1.52	3.67	0.07		
Power	0.17		2.23		1.77	
Exponential	0.26		1.81			2.04



(a) (b) (c) (d)

Figure 2. Fitted curves: (a) linear, (b) linear segment, (c) power and (d) exponential

3.2. Scatter diagrams

The scatter diagrams of the results for the four functions are shown in Figure 3. From the diagrams, the order of differences between y=x line and fitted lines is linear<linear segment<power<exponential, which means the simplest linear function best describes the relation. The slopes are all less than 1, and the order is linear>linear segment>power>exponential. We can see that the observed are larger than the calculated for higher ET rates. However, for the four functions, the points are evenly distributed at the two sides of y=x line, which indicates that all the four functions can effectively describe the relation between groundwater ET rate and depth.

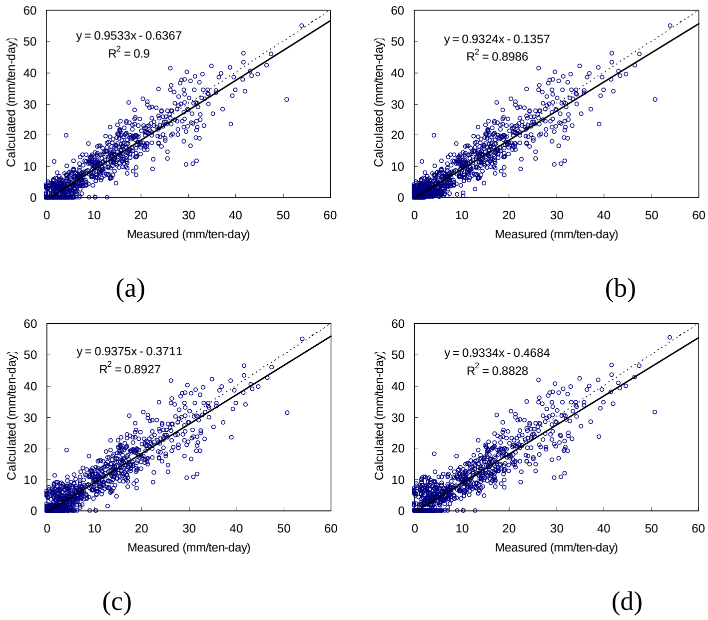


Figure 3. Scatter diagrams of results for the four functions (The dash lines are y=x)

3.3. Statistical comparisons

The results of statistical comparisons for the four ET functions are shown in Table 2. Values of the coefficient of determination (R^2) between the measured and calculated values of groundwater ET rates for all the functions were found to be high (about 0.9; most statisticians consider a coefficient of determination of 0.7 or higher for a reasonable model), which implies about 90% of the variation in groundwater ET is accounted for by the ET functions.

The order of coefficient of determination is linear>linear segment>power>exponential, which is consistent with the order of ET surface depth; however, the order of relative error is consistent with the order of extinction depth. The reason is that in LIS, significant groundwater ET occurs when groundwater depth is less than 1.63 m, the linear function best captures this character; while linear segment yields the largest extinction depth, which captures groundwater ET when groundwater is deep.

Table 2. Results of statistical comparisons for the four ET functions

	$\sum (y_i - f(x_i))^2$ (m^2)	R^2	RE (%)
Linear	12704.4	0.908	15.89
Linear segment	14601.0	0.894	9.15
Power	16009.7	0.884	12.79
Exponential	17804.1	0.871	14.91

3.4. Recommendations for regional groundwater ET modeling

The above results show that there is no significant difference between coefficients of determination for the four functions. We believe that for the Huibei site, all the four functions can effectively describe the relation between groundwater ET and groundwater depth if proper parameter values are chosen. Given the similarity in model predictions, the linear function has only two parameters and the linear segment function has at least 4 parameters for two segments. The choice of which function to be used should be made on the basis of another factors like parameter sensitivity and ease of optimization issues.

Groundwater ET is also dependent on crop and soil types. Several crops, rice, wheat and corn, are grown in LIS, and some lands are fallow during winters. Soil types in LIS include loam and clay. To improve performance of groundwater ET simulation, considerations for these factors are needed.

4. SUMMARY AND CONCLUSIONS

Four groundwater ET functions, linear, linear segment, power and exponential, were compared using lysimeter data from experiments conducted at Huibei Irrigation Experiment Station in China. The results show that values of the coefficient of determination between the measured and calculated values of groundwater ET rates for all the four functions were high, which indicates that for the study site, all the four functions can effectively describe the relation between groundwater ET rate and groundwater depth if proper parameter values are chosen. The linear function is simple and suitable for shallow groundwater tables. The linear segment function requires more parameters while it can effectively simulate ET from deep groundwater. For simulation of regional groundwater ET in LIS, crop and soil types should be taken into consideration.

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