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Testing a Particle Approach to Determine Surface Water Flow Over a Terrain

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Abstract: Validated biophysical simulation models can be applied to undertake an *a priori* assessment of environmental outcomes by taking into account the interaction of various human activities, biological and physical processes. These include land management and practice, water balance, erosion, nutrients, carbon and vegetation dynamics. Model output can be an integral part of an evidence-based approach to the procurement of environmental outcomes.

Biophysical models typically operate on discrete computation cells of a catchment in the order of 20 - 50 metres and on daily time steps. They can be used to evaluate the production and environmental aspects of farming systems and catchments. These models also provide estimates of deep drainage, sub-surface lateral flows and surface runoff. Water partitioning is important as vertically dominated recharge and horizontally dominated flow produce very different environment outcomes. However, the effect of these flows on nutrient, contaminants and sediment transport can only be assessed if the output from one computational cell forms the input to the neighbouring computational cells of the farm/catchment.

There are many single-flow and multiple-flow algorithms reported in the literature to determine surface water flow directions. These include the Deterministic 8 (D8), multiple flow direction, two-directional edge-centred routing, two directional block-centred and D∞ flow path algorithms. The D8 single-flow algorithm essentially directs flow from each cell to one of its 8 neighbouring cells that has the steepest downslope drop. The other algorithms were developed to remove the inability of D8 in routing flow over flats and sinks as well as to provide ways of partitioning flow from a cell to more than one neighbouring cell. The surface flow can also be determined by one of many methods from computational fluid dynamics (CFD). The flow over terrain and over time come directly from the solutions of the Navier-Stokes equations. Unfortunately, large computational resources are required to obtain the necessary spatial and temporal resolutions demanded by these CFD techniques.

The aim of this paper is to report on our effort in developing an algorithm to direct the flow of a single water particle using ideas from particle methods in CFD. In this method, the governing equation is derived from the principle of conservation of momentum. The particle will move according to gravity, attraction and repulsion from the terrain surface. The repulsion is to stop the particle from penetrating the terrain. The motivation here is that our approach will also take into account the particle momentum in its flow and provide temporal information about the flow. If many particles are used simultaneously, their interactions will produce multiple-direction flow.

Keywords: *Particle method; flow direction; catchment modelling.*

1. INTRODUCTION

Flow directions based on digital elevation models (DEMs) are needed in hydrology to determine the flow paths of water and the movement of sediments, nutrients and contaminants. The earliest and simplest method for specifying flow directions is to assign flow from each cell to one of its eight neighbours in the direction with steepest downward slope. This method, designated D8 (8 flow directions), was introduced by O'Callaghan and Mark (1984) and has been widely used. This simple routing method can be quite effective for routing flow over areas of substantial relief with a highly accurate DEM. However, the D8 algorithm fails in areas with sinks and flats. Sinks (or pits) are depression drained areas with no outlet. Flats are areas of level terrain with slope = 0 (from one cell to all neighbouring cells) and flow direction that cannot be determined with reference to its eight neighbouring cells alone.

The limitations of D8 arises from its method of partitioning flow into only one of eight possible directions. Many approaches have been proposed to remove its limitations and this remains an active area of research (Kenny et al., 2008). One early approach, suggested by Fairfield and Leymarie (1991), is to assign randomly a flow direction to one of the downslope neighbours, with the probability proportional to slope. Other early approaches include the multiple flow direction methods of Quinn et al. (1991) and Freeman (1991), the stream-tube approaches of Costa-Cabral and Burges (1994) and the D∞ algorithm of Tarboton (1998). Essentially, these multiple flow algorithms provide ways of partitioning flow from a cell to more than one neighbouring cell. Other workers presented algorithms for routing flow over flat areas and through depressions (see for example, Kenny et al., 2008). All of these algorithms, however, do not consider dams and their retention capabilities. Recently, Schauble et al. (2008) proposed an approach to extend the D8 algorithm to take into account dams and their retention capabilities so that the influence of dams on the dynamics of a river system can be studied.

The detailed modelling of water flow over a terrain can be determined by one of many methods from computational fluid dynamics (CFD). The flow over terrain and time come directly from the solutions of the Navier-Stokes equations. There are two main approaches to fluid flow simulation - Eulerian and Lagrangian. The Eulerian, or grid based, approach relies on regular samples of data throughout the computational domain. Finite difference or finite element methods are used to solve the Navier-Stokes equations and the fluid surface is represented with a level set of an implicit surface function. An alternative to grid methods are Lagrangian methods. These methods track moving particles of fluid through the computational domain. Unfortunately, large computational resource is generally required to obtain the necessary spatial and temporal resolutions demanded by these CFD techniques.

In this paper, a new method for calculating flow directions is presented. The method is developed from the particle method of computational fluid dynamics. It is relatively simple and demands less computational resource than the above mentioned CFD techniques for its application. It has no inherent difficulties in dealing with flats, sinks and dams in the landscape that plague the D8 algorithm and its many extensions mentioned above.

2. PARTICLE MODEL

Here, we propose a new method for finding the flow path of fluid over a terrain that is based on solving the momentum equation for a fluid particle. There are two classes of numerical techniques for fluid flow calculation reported in the literature - grid-based and meshless methods. The number of computational elements required to represent the terrain is generally much larger than the number of fluid elements required to determine the flow paths of water over a terrain. Thus, for the sake of computational efficiency, we will solve our governing equations for fluid flow by meshless method where computations are only needed in those parts of the computational domain that have fluid elements.

Meshless methods have attracted much attention recently. Two distinct directions are followed by these methods. One is based on field approximations such as radial basis functions (RBF) (Kansa, 1990), element free Galerkin and moving least square approximations. The other is based on kernel approximations such as smoothed particle hydrodynamics (SPH) (Monaghan, 1992). These methods have been applied successfully to solve scientific and engineering problems. For example, Ha (2006, 2007) showed that RBF and generalised SPH produce good numerical solutions of high accuracy for a range of problems in computational fluid dynamics. In most of these applications, the governing equations are the Navier-Stokes equations. The numerical solutions of these equations demand relatively large computational resources and require sophisticated algorithms to ensure the computations are stable and accurate.

As fluid elements in the meshless method represent fluid directly, it is particularly advantageous for free surface flow calculation. Another advantage of meshless method over grid based method is in their computer implementation. There is no dimensional difference between 1D, 2D and 3D as far as computer coding for their implementation is concerned. The main drawback of meshless method is the generally longer computational times when compared to grid based method.

Here, we develop and present a new method for finding the fluid flow over a terrain that requires minimal computational resources for its application. In this method, a fluid element is regarded as a particle that has size and mass associated with it. We will develop our particle method from kinematics arguments. The major force that drives the flow of water over a terrain is gravity. The other force we need to consider is the force of interaction between fluid particles and that between fluid particles and terrain particles. Hence the momentum equation for driving the motion of fluid particle i is as follows:

$$m_i \frac{d^2 \boldsymbol{x}_i}{dt^2} = \boldsymbol{F}_i - m_i \boldsymbol{g} \tag{1}$$

where m_i, $\boldsymbol{x}_i$, t and F_i denote mass, position, time and short range force on particle i respectively. The body force $\boldsymbol{g}$ denotes long range force, such as gravity which impacts on all particles uniformly. Here, bold face characters denote 3D vector quantities. To ensure conservation of momentum, the force on particle i by particle j must be equal but opposite to the force on particle j by particle i. Following Greenspan (1997), we model the interaction force $\boldsymbol{F}$ as Lennard-Jones type force.

$$\boldsymbol{F} = \left(\frac{H}{r^q} - \frac{G}{r^p} \right) \boldsymbol{i} \tag{2}$$

where $\boldsymbol{i}$ denotes a unit vector specifying the force direction, $r = |\boldsymbol{x}_i - \boldsymbol{x}_j|$ denotes separation distance between two particles, p and q are positive constants with $q > p$. The quantities G and H are proportional to the size of the fluid and terrain particles. They are chosen such that $|\boldsymbol{F}| = 0$ when the separation distance between two particles equals particle size. The force is repulsive when the separation distance is less than the particle size. The magnitude of repulsion increases exponentially as separation decreases. The repulsive force stops the fluid particle from falling through the terrain surface. It also stops the particles from occupying the same location at the same time ensuring mass conservation. The force is attractive when the separation distance is larger than particle size. In this paper, $p = 1$, $q = 3$ and particle size is the same as the cell size of the DEM. Future work will include more detailed modelling of this attractive force to take into account the effects different ground covers and terrain surfaces have on water flow. We envisage different force terms for different interacting pairs of DEM and fluid particles.

The above formulation results in a system of second order differential equations from a given set of fluid particles and DEM particles that form the terrain surface. The DEM particles are stationary while the fluid particles move according to the forces acting on them that are defined by Equation (1). In this paper, we use the leap frog time stepping scheme to solve the momentum equations. In the following section, we examine the performance of our model for fluid flow over a terrain.

3. EXAMPLES

To examine the effectiveness of the model presented in the previous section for determining water flow over a terrain, we first apply it to a synthetic terrain the coordinates (x, y, z) of which are given by the following function:

$$z = \left[x - \left(1 - \frac{1}{2}\sin(y)\right)\right]^2 + y, \qquad x \in [0,2], \quad y \in [0,4]. \tag{3}$$

This terrain slopes downwards linearly from north to south and it increases in height quadratically to the east and to the west of a point that departs from $x = 1$ sinusoidally from south to north. It is expected that water from any point on the terrain will flow towards the low point of the terrain. Figure 1 shows the flow paths for line sources at 4 different y values near the top end of the terrain. Initially, the fluid particles are slightly above the terrain surface. They fall under gravity as if they were rain drops falling on the landscape. In these examples, the number of fluid particles used is 49 and a scale factor of 1,000 has been applied to the coordinates defined by Equation (3). The terrain is colour-shaded according to its elevation with high and low values represented by colours in the red and blue end of the spectrum respectively (see the colour bar for detail). The flow paths of water are shown in white lines. It is clear that, in all cases, the water eventually flow down to the lowest part of the terrain for each latitude as expected. This confirms that our particle method can predict the flow paths of water over the landscape.

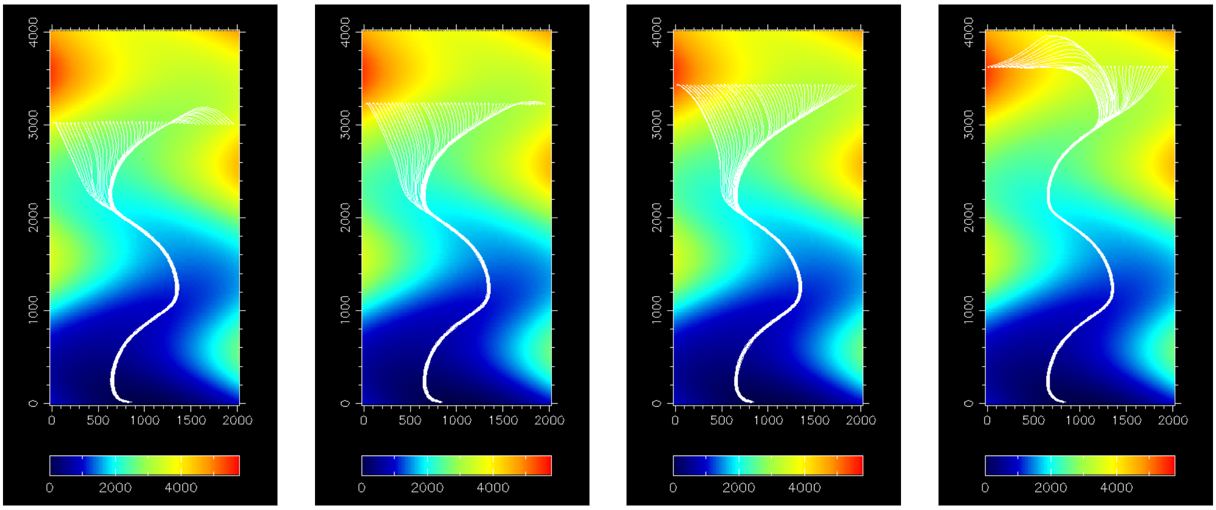

Figure 1: Flow paths from line sources at y = 3020, 3220, 3420 and 3630 m over a terrain defined by Equation (3).

Next, we compute the flow paths of water over a DEM of the Avon-Richardson catchment (371,000 ha) located in the north-central Victoria for different sets of initial fluid particles (see Figure 2 for a map of the region). The DEM is made up of regular 50 × 50 m^2 cells. In all the figures in the remainder of this section, the circles in magenta represents the positions of the source particles. The terrain is again colour-shaded according to its elevation with high and low values represented by colours in the red and blue end of the spectrum respectively. Note that the high region of the catchment is in the south-eastern part of the terrain. Initially, the fluid particles are slightly above the terrain. They fall under gravity and flow down the terrain. In Figure 3, the flow patterns from 32 fluid particles are shown at 4 time instances in order to show the flow of water with time. The figure also compares the results with and without enabling the fluid particles to interact with one another according to $\boldsymbol{F}$ defined by Equation (2). It would be the case without interaction among the fluid particles when the fluid particles at different source locations are released at different times such that they are the only particles within their interaction distance on their way down the terrain. The figure shows that water can also flow uphill even though the overall flow is predominantly going downhill. For example, the flow originating from the red part of the DEM moves alternatively from high region to low region and then from low region to high region as it zig-zags down the catchment. Water can move up slope when it has sufficient momentum gained from its movement down the opposite slope of a valley. The crossing of flow paths indicates a point in the landscape receiving flows from different streams that have different origins and have travelled through different parts of the catchment. The different streams from different origins may arrive at the same location in the catchment at different times. We expect the flow patterns would also be different

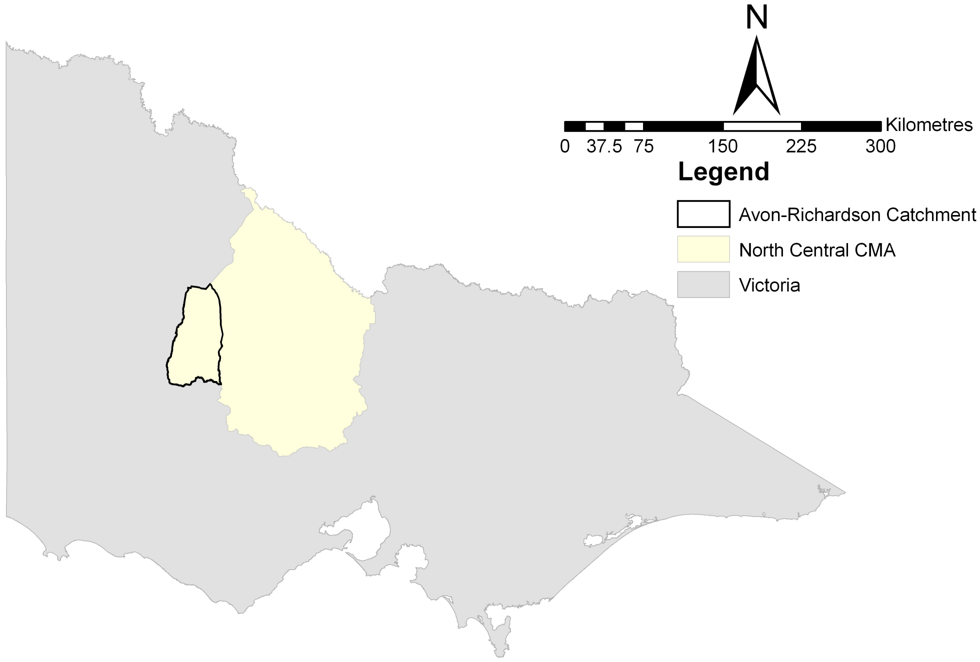

Figure 2: Location map of the Avon-Richardson Catchment.

for different sets of source points. However, it would still be expected that the flow would eventually flow towards the lower reaches of the catchment. This is indeed the case from simulation results presented in Figures 4 and 5. In these examples, the different sets of line sources are at different distances from the lower eastern edge of the catchment. In these cases, the number of fluid particles used is 38. Figure 4 shows the flow patterns reached at four different time instants from the same set of source particles. Figure 5 shows the overall flow patterns for different sets of source points from those used in the last two figures. In all these examples, our particle model is able to predict reasonable flow patterns. The flow pattern shows that the flow is not always going downhill. The flow could go uphill in some parts of its pathway down the catchment until gravity becomes greater than the upward momentum again.

4. CONCLUDING REMARKS

The numerical examples presented in the previous section demonstrate the potential effectiveness of our particle model and its implementation as an alternative approach for finding the flow of water over a terrain. The particle method is developed from the physical principle of momentum conservation. It has no difficulty in handling flats, sinks and dams. The water flow over these parts of the landscape is determined by the momentum the particle has from its passage over the terrain. In addition, our particle approach produces results that are not available from D8 type approaches. For example, they could not account for flow going uphill as these algorithms only assign flow direction to downslope neighbours. Furthermore, D8 type algorithms do not predict flows from different source locations may arrive at a point in the catchment at different times. This may have implications for the way we model biophysical processes in a catchment and they are topics for future research. For example, the arrival of different amount and composition of nutrients and contaminants at different times during a crop's life cycle may affect its growth pattern and yield. Future works will also include validation of our particle method against observations in nature.

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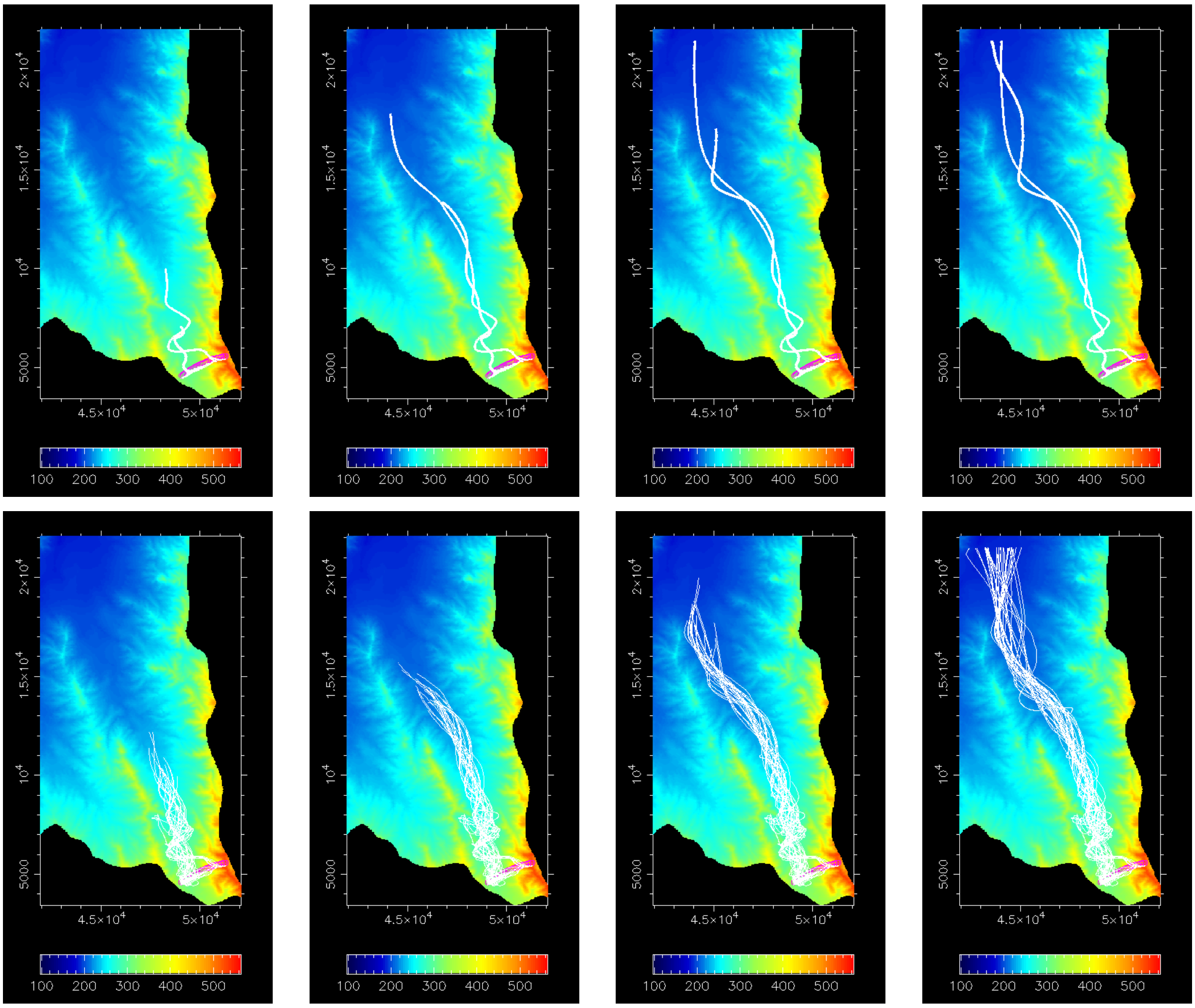

Figure 3: Water flow over the Avon-Richardson catchment at different times with (top row) and without (bottom row) fluid interaction switched on.

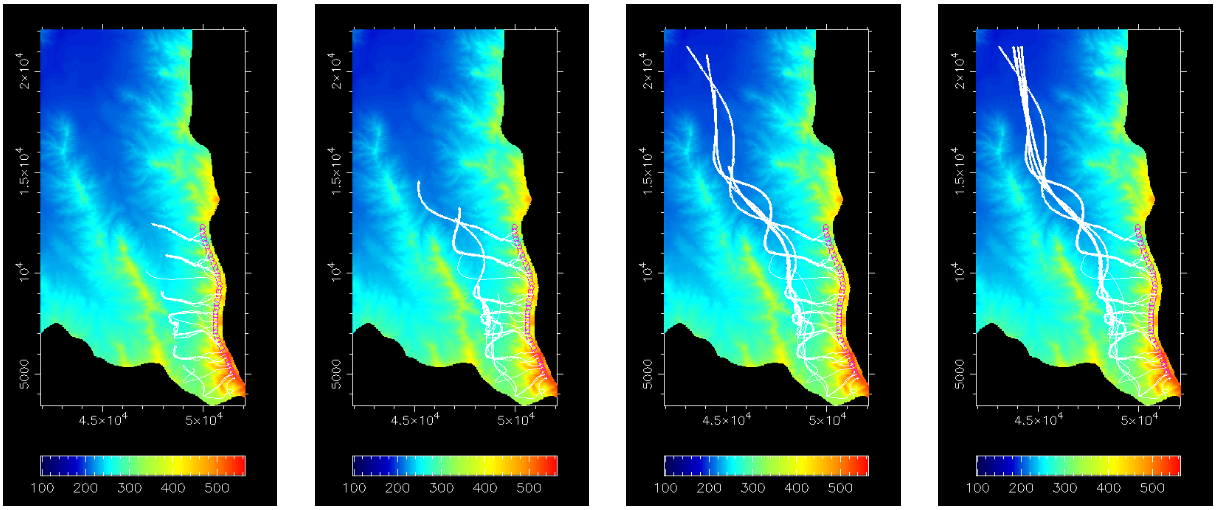

Figure 4: Flow patterns over the Avon-Richardson catchment at different times.

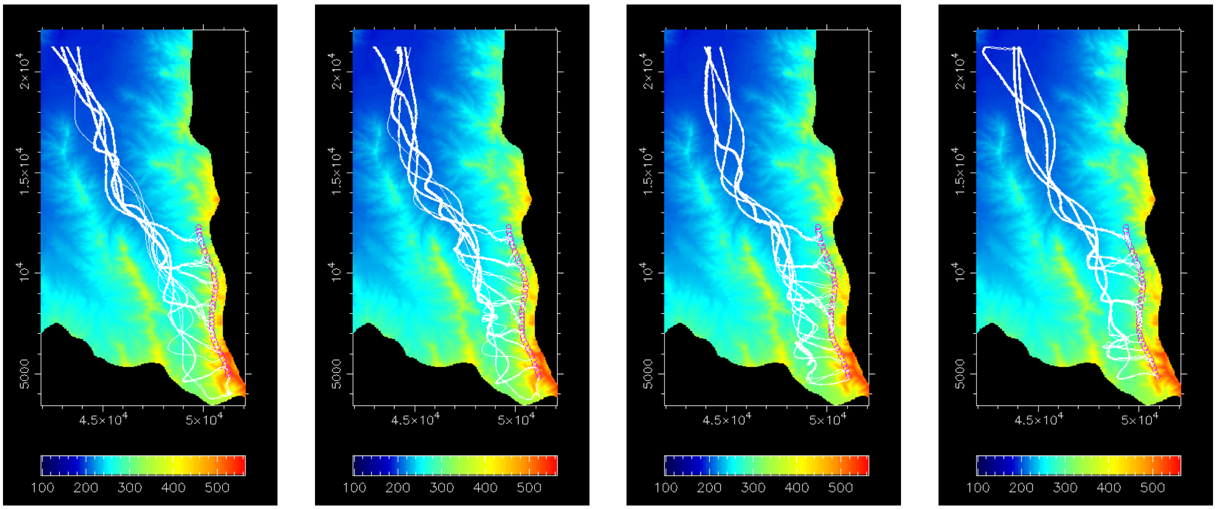

Figure 5: Flow patterns over the Avon-Richardson catchment from different source locations.

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