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A Flexible Stochastic Approach to Constraining Uncertainty in Forward Stratigraphic Models

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Abstract: Pinpointing the location of reservoir rocks and stratigraphic traps is crucial to the cost-effective extraction of oil. Forward stratigraphic models have been used to aid interpretations of these rocks and traps by determining their potential sand, shale and silt content. These stratigraphic models often rely on diffusion as a basis for simulating the processes of rock-erosion and the deposition of sediment, which forms the basis for rock formation. Diffusion-based modelling is consistent with observations in sedimentary environments over long time-scales. However, a persistent problem with these models is the determination of coefficients that specify the effectiveness of diffusion for each sediment type.

These coefficients, the so-called transport coefficients, are usually determined by a manual inversion to fit the data. In many cases, varying the transport coefficients by orders of magnitude can still achieve consistency with observations of sediment thickness from seismic data. By mapping the uncertainty in these transport coefficients to probability distributions, a probabilistic method can be used to determine the uncertainty in the output of the model, the basin bathymetry and the location of rocks and their original sediment compositions. We provide an example from the Ebro Basin, located off the east coast of the Iberian Peninsula.

The stochastic approach consists of generating a surface response of the stratigraphic forward model for each point in the domain. The building blocks, in our case, are Hermite polynomials, an orthogonal set. The surface response is a function of the uncertain parameters in the stratigraphic model, the most prominent of which are the diffusion coefficients. Assigning probability distributions to the uncertain parameters then determines the statistical properties of the surface response model. We show the time-dependent development of uncertainty in the Ebro Basin in the sand, silt and shale content of deposited sediment using this methodology.

Keywords: *Stratigraphy, numerical modelling, stochastic, uncertainty*

1. INTRODUCTION

Numerical modelling of stratigraphy can aid in the understanding of the evolution of strata and their composition. Diffusion-based models that average the effects of several physical processes have been successful at replicating stratigraphic structures and are used today in the oil industry and academia (Paola, 2000). Underlying these codes is the assumption that on the long term, sediment moves based on slope and its material type, the latter given by specifying a transport coefficient or efficiency for the material.

Transport efficiency is an abstract concept that is not physically measurable. For each given basin, material will be transported more or less efficiently based on the type and number of physical processes acting. In the near shore, sediment is reworked by waves, while on the marine shelf, wave and current energy may lift the material and cause it to be deposited elsewhere. On the continental slope and on the basin floor, deposition occurs because suspended sediment is primarily transported by mass wasting and gravity currents.

The lack of measurability of the transport efficiency for each material can only be determined by matching the data available in a basin. One such technique is using automated inversion; equations for inversion have been recently worked out for a two-material system for one time layer (Schroll, 2009, submitted). However, diffusion-based numerical codes often calculate multiple physical processes. A transport coefficients can exist for each material based on gravity, water and wave driven diffusion as well as parameters for along-shore currents, carbonate production, compaction and sea-level, to name a few.

Because we wish to capture this complexity, we can use a stochastic approach to diffusion-based stratigraphic models (Clark *et al.*, 2009, in prep.). The idea is to treat the most uncertain or difficult to measure parameters, usually the transport coefficients, as stochastic variables rather than fixed quantities. For our case, we treat a stochastic variable as spanning a probability density function, either because the range of value is uncertain, or because, in practice, the sample of grain-sizes that represent, for example, sand approximates such a distribution.

Because some of the initial parameters can be treated as stochastic, the output function can also be seen as stochastic, with an associated probability density function. How can the output statistics be determined? One approach would be to use a Monte Carlo method, running the solver with many combinations of parameter values selected according to each parameter's probability density function. Typically this might require thousands of calculations with different parameter values to achieve the required accuracy; a process that might take a month to achieve with the simplified example case we deal with below. For other applications, the stochastic method we employ has been verified with the Monte Carlo method (Li & Zhang, 2007; Loeven *et al.*, 2007). We instead verify the method's applicability to stratigraphic modelling, a case with many variables and long solution times.

2. THE EBRO DELTA AND MARGIN

To demonstrate the PCM with regard to a particular setting, we start with a model of the Ebro margin (Figure 1), in which the diffusion parameters have been manually tuned to produce stratigraphic characteristics comparable with those observed in seismic observations. Using Dionisos, a diffusion-based package for stratigraphic modelling (Granjeon, 1997; Granjeon & Joseph, 1999), we model progradation of the Ebro Delta since 3 Ma. We model a 250km x 250km section of the Delta (red box, Figure 1), taking the initial bathymetry as smoothed present-day seafloor. Sediment volumes entering the computation domain are derived according to the input volumes of Nelson (1990). We model three material types in the basin (sand, shale and silt) and calculate their fraction of volume, averaged for each 0.5 Myr layer. In addition, the model simulates subsidence rates in the order of 30 m/Myr on the outermost shelf linearly reduced to 0 m/Myr on the inner shelf.

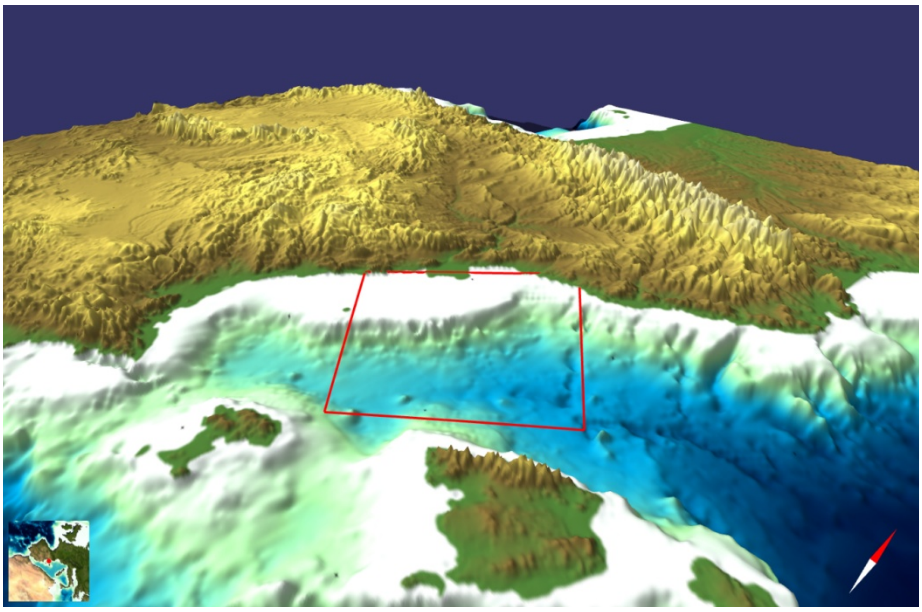

Figure 1. Ebro margin topography as viewed from the southeast. The study area is outlined by the red box. A compass and regional map are inset for reference. The image was produced by 4D Lithosphere Model.

3. METHODOLOGY

We use the Probability Collocation Method (Tatang, 1994; Tatang *et al.*, 1997) for determining the statistics of the stratigraphic modelling. The method uses a polynomial expansion of model parameters to represent the response of the numerical stratigraphic model. By choosing polynomials that are orthogonal to each other (Ghanem & Spanos, 1991), the statistical properties of the expansion can be calculated from the model parameters and such an expansion is known as a Polynomial Chaos Expansion (Ghanem & Spanos, 1991). Representing the stratigraphic model as a function f, dependent on a number of input parameters ω, then the model can be approximated by the following expansion:

$$f(\omega) \approx \sum_{i=0}^{M-1} c_i \Phi_i \left(\xi(\omega)\right) \tag{1}$$

In this representation, Φ_i are a set orthogonal functions, c_i are a set of coefficients to be determined and $\xi(\omega)$ are the probability density functions of each parameter. In (1), there are M coefficients, $c_0 \ldots c_{M-1}$ to be determined. We therefore need to compute the model, f, M times, each time with a unique set of parameter combinations. The Probability Collocation Method (PCM) chooses these parameter combinations from the roots of the orthogonal set of polynomials of 1 higher order than the set used in the expansion.

To test the PCM with regard to stratigraphic modelling, we choose a second order expansion of six diffusion parameters, modelled as stochastic parameters (Table 1). By mapping the three log-normally distributed parameters to normal distributions, we can select Hermite polynomials as the orthogonal set for the Polynomial Chaos Expansion. The following are the Hermite polynomials of second order or lower:

$$\Gamma_0(x) = 1$$
$$\Gamma_1(x) = x$$
$$\Gamma_2(x_i, x_j) = \begin{cases} x_i x_j, & i \neq j \\ x_i^2 - 1, & i = j. \end{cases}$$

A second order expansion therefore has the form

$$f \approx \hat{f} = c_0^0 + \sum_{i=1}^{6} c_i^1 \Gamma_1(x_i) + \sum_{i=1}^{6} c_i^2 \Gamma_2(x_i) + \sum_{i_1=2}^{6} \sum_{i_2=1}^{i_1-1} c_{i_1,i_2}^{1,1} \Gamma_2(x_{i_1}, x_{i_2}) \qquad (2)$$

in which c_m^l are the coefficients, m is a numbering of the coefficients, while l indicates the order of the corresponding Hermite polynomial.

Table 1. Statistical distributions of transport coefficients for water-driven transport for each material type: sand , silt and shale. Log-Normal ~ (μ, σ^2) indicates a lognormal distribution with an underlying normal distribution with mean μ and standard deviation σ .

		Material Type		
Region	**Statistical Property**	**Sand**	**Silt**	**Shale**
Subaerial	Median Value (km^2/kyr)	100	1000	2000
	Range of Values (km^2/kyr)	0.1 – 5000	10 – 5x10^4	0.1-5
	Distribution	Log-Normal ~ (4.6, 3.5^2)	Log-Normal ~ (6.9, 2.3^2)	Log-Normal ~ (7.6, 2.3^2)
Submarine	Median Value (km^2/kyr)	0.01	0.05	1
	Range of Values (km^2/kyr)	1x10^{-4} – 0.1	5x10^{-4} – 0.5	0.1-5
	Distribution	Log-Normal ~ (-4.6, 2.3^2)	Log-Normal ~ (-3, 2.3^2)	Log-Normal ~ (0, 1.2^2)

Such an expansion has 28 terms, requiring the same number of runs of the stratigraphic model. We select these runs from the roots of the next highest order Hermite polynomial, the third order polynomial with roots $0, \sqrt{3}$ and $-\sqrt{3}$. Following Tatang (1994), we calculate the expectation of the approximation by using the zeroeth coefficient of the expansion, as $E(\hat{f}) = c_0$. Using the orthogonality of the polynomials, and the fact that

$$\sigma(\hat{f}) = \sqrt{E(\hat{f}^2) - E^2(\hat{f})},$$

we derive the standard deviation as the product of the second-order non-identical terms of the Polynomial Chaos Expansion:

$$\sigma(\hat{f}) = \sqrt{\sum_{i_1=2}^{6} \sum_{i_2=1}^{i_1-1} c_{i_1,i_2}^2}$$

4. RESULTS

We calculate statistical moments of the model outputs, the main ones being the basin bathymetry and the sediment concentrations for each calculated depositional layer. For our purposes, we will only show results of the mean and standard deviation. In Figure 2, we show the standard deviation of the basin bathymetry (negative values being below sea-level) at six stages of the model development, separated by 50,000 years. High uncertainty in the bathymetry (green regions) which develops on the top-slope and then propagates down to the basin as sediment fills the basin from approximately 1300m depth to 1000m depth.

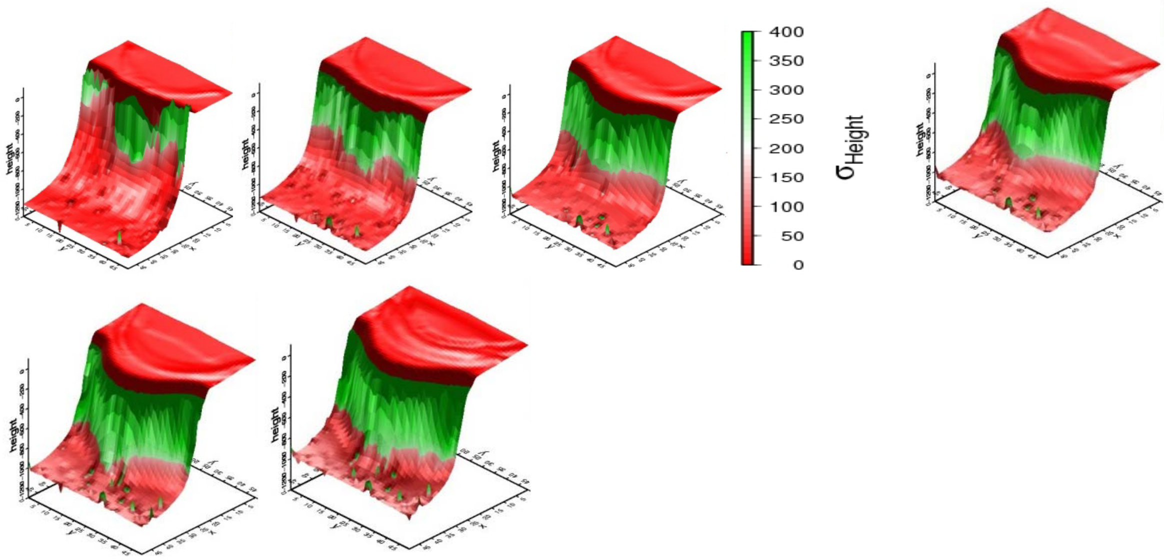

Figure 2. Shaded standard deviation of the basin height of different stratigraphic layers, overlain on the mean basin height for that layer (sea level at 0m), progressing left to right, then top to bottom, the layers are aged 2.5 Ma to present in 0.5 Myr intervals.

The sand fraction shows low levels of deviation in most layers, though with the sea-level low-stand the volume of sand in the layers of 2.0 Ma and 1.0 Ma shows considerable uncertainty and hence dependency on the diffusion coefficients (Figure 3). At other points, the standard deviation varies between 0.0 and 0.2 of the sand fraction.

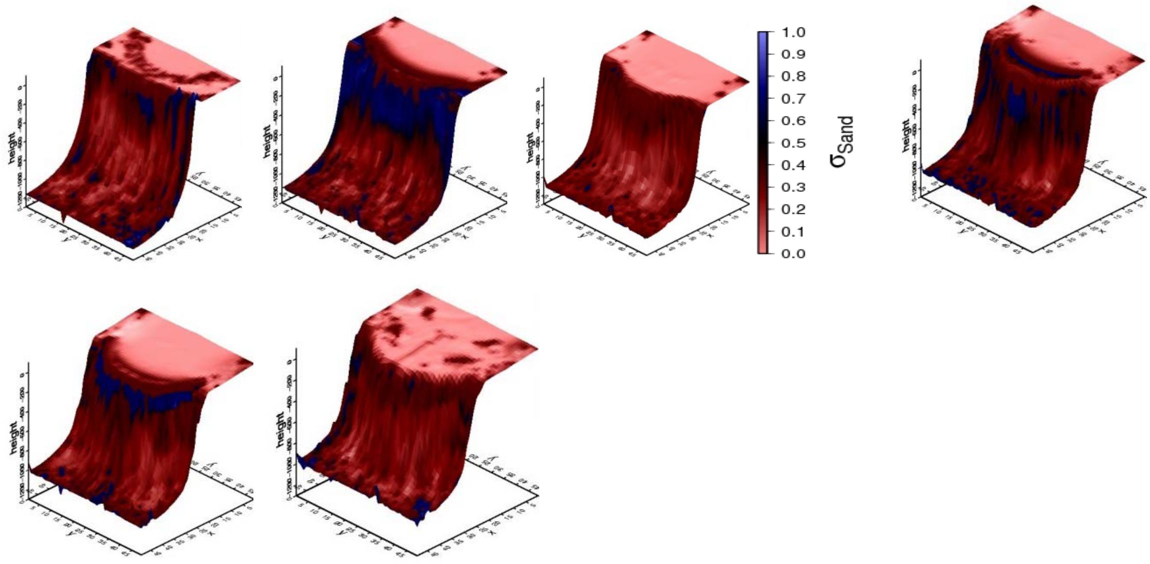

Figure 3. Shaded standard deviation of the sand fraction overlain on the mean basin height, progressing as in Figure 2.

The silt fraction is less uncertain, with a lower standard deviation than the sand fraction, of between 0.0 and 0.1, in general (Figure 4) and far less dependent on sediment influx than sand.

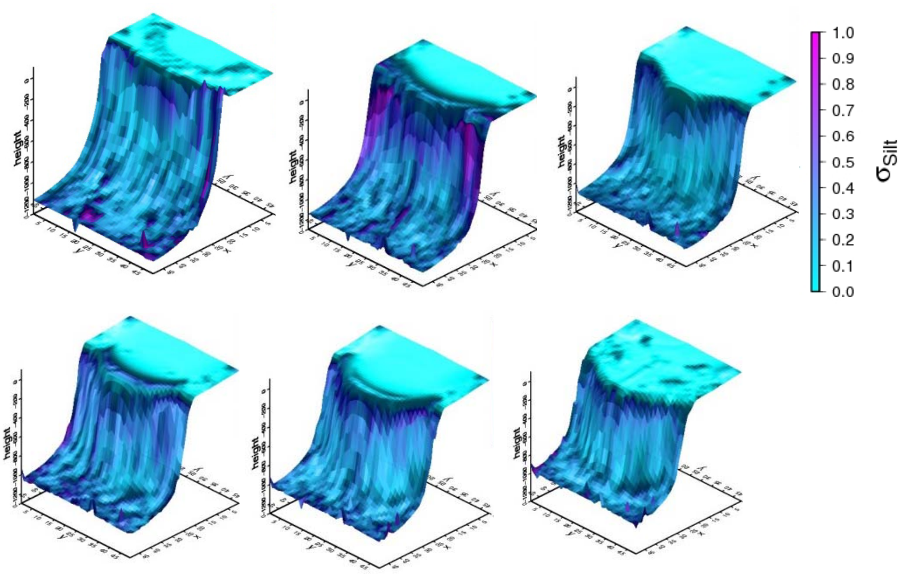

Figure 4. Shaded standard deviation of the silt fraction overlain on the mean basin height, progressing as in Figure 2.

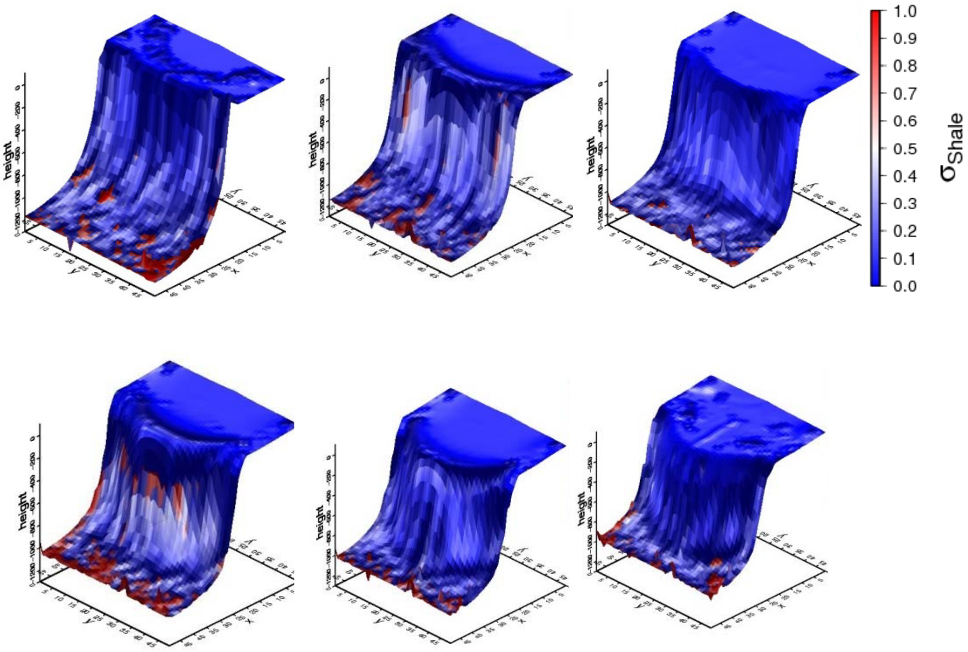

Figure 5. Shaded standard deviation of the shale fraction overlain on the mean basin height, progressing as in Figure 2.

The shale fraction has the greatest uncertainty floor, as the shale fraction is lowest in this region and small changes in the diffusion parameters lead to high jumps in the fraction of shale.

5. CONCLUSIONS

This method provides a flexible way of calculating the effect of uncertainty inherent in the definition of diffusion coefficients on the output measurements of the model. By approximating the model with a Polynomial Chaos Expansion of input parameters, the output statistical moments, such as the mean and standard deviation, can be calculated. The method can be placed around any diffusion-based stratigraphic model without knowledge of the underlying differential equations or numerical method. Thus the method presented here can be flexibly applied to either other numerical models or with other physics, such as carbonate production.

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