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Analysis of Queuing Scheduling Linkage Model to Minimize the Hiring Cost of Machines/Equipments

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Abstract: Hiring of machines/equipment is growing as the most preferred alternative method for financing the latest technology in many developing countries due to budgetary constraints faced by most of industries. The present paper is an attempt to develop an algorithm to minimize the utilization time of machines/equipments taken on rent and hence their hiring cost for the proposed queuing scheduling linkage model. The phase I of service (network of queues) consists of parallel and biserial servers in which the arrival and the service pattern both follows poisson's law. Further, the completion time of jobs in getting phase I of service will be the setup time for first machine in phase II of service consisting of '*m*' machines/equipments in series taken on rent. In industries men, machines, materials and money are involved for production. The manager of an industry is interested to use them in an economic manner so that the cost associated with the production is not increased or it minimum in the competition market. In industry there are some very complicated machines that are often necessary for performing a variety of procedures. In order to stay competitive, up to date equipments are required and buying all of these equipments get really expensive over time. There are industries who take various machines/equipment on rent and hiring of equipment is an affordable and quick solution for many industries, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting of equipments/machines enable saving working capital, give options for upgradation to new technology. The objective of paper is to find the latest time at which machines/ equipments should be hired so as to minimize utilization time and hence their hiring cost with minimum total elapsed time and with minimum mean queue length for the proposed queuing scheduling linkage model. A numerical illustration is also carried to test the efficiency of the proposed algorithm.

Keywords: *Flowshop scheduling, Mean queue length, Utilization time, Waiting time, Hiring Cost*

1. INTRODUCTION

Waiting lines or queues are a common occurrence in everyday life and in many industrial situations. We all observe queues in communication systems, voice or data traffic queues for the transmission, in manufacturing system with several work stations, units completing work in one station wait for access to the next, patients arriving at hospital or a clinic for treatment etc. The amount of time that a nation's population wastes by waiting in queues is a major factor in both the quality of life there and the efficiency of the nation's economy. Queuing theory uses mathematical tools to predict the behaviour of queuing system consisting of a stream of arriving customers, a queue and a service stage. Prediction deals with mean length of queue, mean waiting time, total completion time and so on. Also, the scheduling models concerned with determination of an optimal sequence in which various jobs/customers are served in order to minimize the total elapsed time and some other measure of performance. Scheduling problems exist whenever there is an alternative choice in which a number of jobs/tasks can be performed. In this paper the processing of jobs/customers through a linkage network of queues consisting of biserial and parallel servers linked to a common server with a flowshop scheduling system having '*m*' machines/equipments in series taken on rent is considered. The practical situations of the proposed work can be taken as examples in medical industry, construction industry and manufacturing industry etc. In hospitals, the patients arriving have to be passed through a series of servers / channels / stages before being attended by a doctor/ expert and there after they have to be processed through a number of hired equipments/machines in series. Because of growing tendency for emergency cases, the hospitals have been experiencing a counting increase in the number of patient visits every day. As a result, it has become quite common for the patients arriving during peak usage hour have to wait until it is there turn to be treated by the doctor. Also, the medical science can save the patient's life but proper care leads to a faster recovery. Care giving techniques often require hi-tech up-to date medical equipments and most of these equipment's are expensive. Further, they are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them. Rental of medical equipment/machine is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds. Global Industry Analyst (GIA) announces that the global market for medical equipment rental and leasing is forecast to reach \$ 56 billion by the year 2017. Major factor propelling market growth include growing incidence of chronic diseases, the demand for diagnosis, technological advancements, leading to the obsolescence of old equipments , surging prices of medical equipments and need to curb healthcare expenses. As another example, in construction industry rental of equipment offers contractors and construction companies a cost effective alternative to owing equipments because equipments become a fixed cost without the accompany overhead of ownership such as maintenance, repair and storage. The rental company handles and pays for the warranty, maintenance and up keep the equipment. The worldwide construction equipment rental market is projected to reach an estimated \$193 billion by 2017.

The rest of paper is organized as follows: Section 2 deals with review of existing literature. Section 3 introduces the mathematical model and analysis of proposed linkage model with some basic definitions, various symbols and fundamental theorems required in the progress of the paper. Section 4 describes the algorithm proposed to optimize various characteristics of the proposed model. Section 5 presents a numerical illustration to test the efficiency of the algorithm proposed. The paper is concluded in section 6 followed by the references.

2. LITERATURE REVIEW

The origin of queuing theory can be tracked back to early in the last century when Erlang (1909), a Danish engineer, applied this theory extensively to study the behavior of telephone networks. Jackson (1954) studied the behaviour of a queuing system containing phase type service. Little (1965) gave the formula for calculating the mean queue length. Maggu (1970) introduced the concept of bitendom in theory of queues. Later on various ideas were developed by many researchers with different modifications and argumentations. Singh et al. (2005) studied the transient behaviour of a queuing network with parallel bi-series queue linked with a common channel. Gupta et al. (2011) studied steady state behaviour of a queue model with biserial servers and parallel servers linked in series with a common server. Also, one of the earliest results in flow shop scheduling theory is an algorithm by Johnson's (1954). Smith (1956) whose work is one of the earliest considered minimization of mean flowtime and maximum tardiness. Bagga (1968) studied the sequencing of jobs in flowshop scheduling when the machines are taken on rent. Bagga and Bhambani (2000) obtained an optimal sequence for minimizing the bicriteria for the general *n* x *m* flowshop problem. Sharma and Gupta (2011) minimized the rental cost under a specified rental policy in two stage flowshop. Gupta, Sharma and Gulati (2011) studied the bicriteria in three stage flowshop scheduling to minimize the rental cost of machines with minimum makespan.

The literature reveled that a lot of research work has already been done in the field of Queuing and Scheduling theory individually. Only some efforts have been made to establish a linkage between these two fields of optimization. Singh and Kumar (2009), Maggu and Gupta (2007), Gupta et al. (2012a, 2012b, 2012c, 2012d) have made some efforts to optimized the total flow time, waiting time and service time in Queuing-Scheduling linkage model. Recently Gupta et al. (2013) have established a linkage model of network of queues with flow shop scheduling system including independent transportation time. This paper establishes link between network of queues given by Gupta et al. (2011) with a flowshop scheduling having a system of equipments/machines taken on rent to minimize their hiring cost with minimum makespan as given by Sharma and Gupta (2012d).

3. MATHEMATICAL MODEL AND ANAYSIS

The entire queue model is comprised of three service servers S_1, S_2 and S_3. The server S_1 consists of two

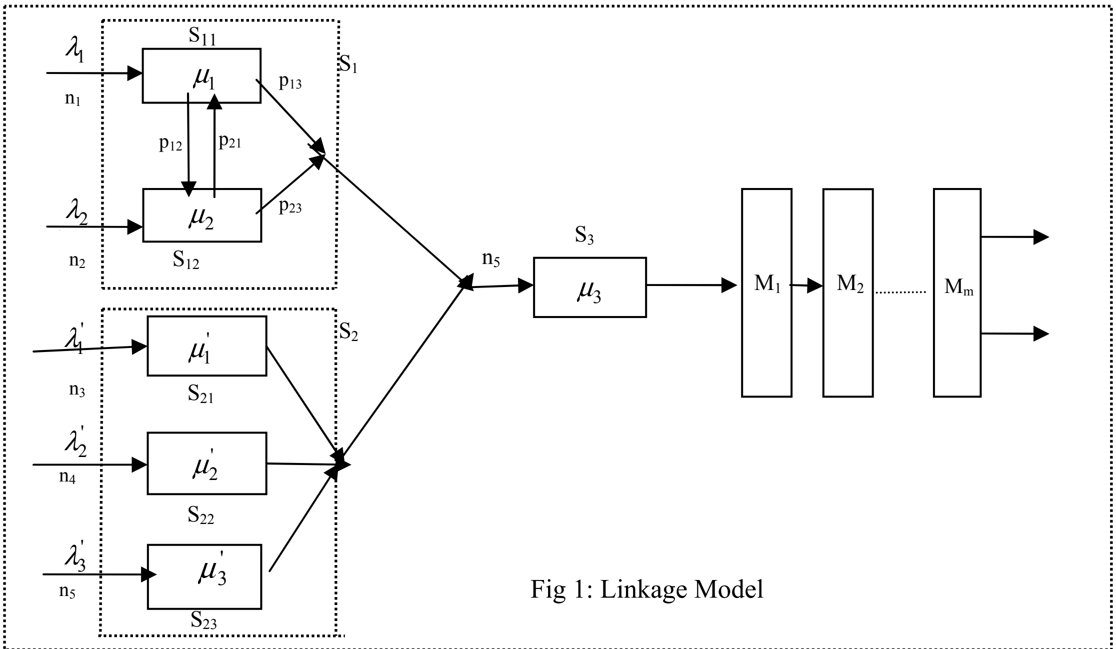

Fig 1: Linkage Model

biserial service sub-servers S_{11} and S_{12}. The server S_2 contains three parallel sub-servers S_{21}, S_{22} and S_{23}. The service server S_3 is commonly linked in series with each of two servers S_1 and S_2 for competition of the phase I of service demanded either at the server S_1 or S_2. The service time at the servers $S_{11}, S_{12}, S_{21}, S_{22}, S_{23}$ and S_3 are distributed exponentially with mean service rate $\mu_1, \mu_2, \mu_1', \mu_2', \mu_3'$ and μ_3 respectively. Queues are said to be formed in front of the service servers if they are busy. Customers arriving at the rate λ_1 after completion of phase service at S_{11} will join server S_{12} or S_3 .i.e. either processed through the network of servers $S_{11} \rightarrow S_{12} \rightarrow S_3$ or $S_{11} \rightarrow S_3$ with the probabilities p_{12} or p_{13} such that $p_{12} + p_{13} = 1$ and those arriving at the rate λ_2 after completion of phase service at S_{12} will join sever S_{11} or S_3 .i.e. either processed through the network of servers $S_{21} \rightarrow S_{11} \rightarrow S_3$ or $S_{21} \rightarrow S_3$ with the probabilities p_{21} or p_{23} such that $p_{21} + p_{23} = 1$. The customers arriving at the rates λ_1', λ_2' and λ_3' follow the network of servers $S_{21} \rightarrow S_3$, $S_{22} \rightarrow S_3$ and $S_{23} \rightarrow S_3$. After coming out from the server S_3 .i.e. the Phase I of service, customers/jobs has to be processed through a system of equipments/machines (Phase II of service) taken on rent.

The following notations have been used in processing of jobs through the system of equipments/machines:

S : Sequence of jobs obtained by applying Johnson's procedure

M_j : Machine j, j= 1, 2, 3… m

$A_{i,j}$: Processing time of i^{th} job on machine M_j

$L_j(S)$: The latest time when machine M_j is taken on rent for sequence S

$t_{i,j}(S)$: Completion time of i^{th} job for sequence S on machine M_j

$t'_{i,j}(S)$: Completion time of i^{th} job for sequence S on M_j when M_j start processing jobs at time $L_j(S)$

$I_{i,j}(S)$: Idle time of machine M_j for job i in the sequence S

$U_j(S)$: Utilization time for machine M_j

$R(S)$: Total minimum rental cost of machines for the sequence S of jobs

C_j : Rental cost for machine M_j (j = 1, 2, 3… m) per unit of time.

Let $P_{n_1,n_2,n_3,n_4,n_5,n_6}$ be the joint probability that there are n_1 units waiting in queue Q_1 in front of S_{11}, n_2 units waiting in queue Q_2 in front of S_{12}, n_3 units waiting in queue Q_3 in front of S_{21}, n_4 units waiting in queue Q_4 in front of S_{22}, n_5 units waiting in queue Q_5 in front of S_{23} and n_6 units waiting in queue Q_6 in front of S_3. Now proceeding on the lines of Gupta et al. (2011), the joint probability function for the proposed queue network (Phase I of service) is $P_{n_1,n_2,n_3,n_4,n_5,n_6} = \rho_1^{n_1}\rho_2^{n_2}\rho_3^{n_3}\rho_4^{n_4}\rho_5^{n_5}\rho_6^{n_6}(1-\rho_1)(1-\rho_2)(1-\rho_3)(1-\rho_4)(1-\rho_5)(1-\rho_6)$

Here, $\rho_1 = \dfrac{(\lambda_1+\lambda_2 p_{21})}{\mu_1(1-p_{12}p_{21})}$, $\rho_2 = \dfrac{(\lambda_2+\lambda_1 p_{12})}{\mu_2(1-p_{12}p_{21})}$, $\rho_3 = \dfrac{\lambda_1'}{\mu_1'}$, $\rho_4 = \dfrac{\lambda_2'}{\mu_2'}$ $\rho_5 = \dfrac{\lambda_3'}{\mu_3'}$ and

$$\rho_6 = \left[\ \frac{\lambda_1'+\lambda_2'+\lambda_3'}{\mu_3} + \frac{p_{13}(\lambda_1+\lambda_2 p_{21}) + p_{23}(\lambda_2+\lambda_1 p_{12})}{\mu_3(1-p_{12}p_{21})}\ \right].$$

Also, the mean queue length (average number of customers/jobs) = L = $L_1 + L_2 + L_3 + L_4 + L_5 + L_6$

Where, $L_1 = \dfrac{\rho_1}{1-\rho_1}$ $L_2 = \dfrac{\rho_2}{1-\rho_2}$, $L_3 = \dfrac{\rho_3}{1-\rho_3}$, $L_4 = \dfrac{\rho_4}{1-\rho_4}$, $L_5 = \dfrac{\rho_5}{1-\rho_5}$ and $L_6 = \dfrac{\rho_6}{1-\rho_6}$.

The average waiting time of customers/jobs as given by Little's Law is $W = L/\lambda$.

3.1. Definition

Completion time of i^{th} job on j^{th} machine is defined as:

$t_{i,j} = max\ (t_{i-1,j}\ ,\ t_{i,j-1}) + A_{i,j}$ for $j \geq 2$. Also, $t_{i,j} = \max(t'_{i,j-1}, t'_{i-1,j}) + A_{i,j}$.

Completion time of i^{th} job on j^{th} machine when M_j starts processing jobs at the latest time L_j is denoted by $t'_{i,j}$ and is defined as, $t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j}$. Also, $t'_{i,j} = \max(t_{i,j-1}, t'_{i-1,j}) + A_{i,j}$

Further, the machines will be taken on rent as and when they are required and will be returned as and when they are no longer required. .i.e. the first machine will be taken on rent in the starting of the processing the jobs, the second machine will be taken on rent at time when the first job is completed on the first machine and so on continuing in this way, the m^{th} machine will be taken on rent when the first job is completed on the $(m-1)^{th}$ machine.

3.2. Theorem: Let '*n*' jobs J_1, J_2, J_3… J_n are processed through '*m*' machines M_j (*j* = 1, 2… m) in order M_1-M_2-M_3-…….-M_m with no passing allowed. If $A_{i,j}$ represents the processing time of i^{th} job (*i* =1, 2, …, n) on j^{th} machine (*j* = 1, 2, …., m) such that $\min A_{i,s} \geq \max A_{i,(s+1)}$; *s* = 1, 2, …, (*m*-2), then the optimal schedule minimizing the total elapsed time is given by the following decision rule: Job J_k proceeds job J_{k+1} if $\min\{G_k, H_{k+1}\} < \min\{G_{k+1}, H_k\}$;Where, $G_i = A_{i,1} + A_{i,2} + \text{----} + A_{i,(m-1)}$ and $H_i = A_{i,2} + A_{i,3} + \text{----} + A_{i,m}$.

3.3. Theorem: The processing of the jobs on m^{th} machine M_m at the latest time L_m= $\sum_{i=1}^{n} I_{i,m}$ keeps the total elapsed time $t_{n,m}$ remain unaltered.

3.4. Theorem: The processing of jobs on j^{th} machine M_j at the latest time $L_j = \min_{1\leq k\leq n}\{Y_{kj}\}$ keeps the total elapsed time $t_{n,m}$ remain unaltered, where $Y_{1j} = L_{j+1} - A_{1,j}$ and $Y_{kj} = t'_{(k-1),(j+1)} - \sum_{i=1}^{k} A_{i,j}; k>1; j=(m-1),(m-2),.....,3,2.$

4. PROPOSED ALGORITHM

The following algorithm is proposed to find the optimal sequence of the jobs processing with minimum makespan, the latest time at which equipments/machines should be taken on rent, minimum mean queue length of jobs processing, when the completion time (waiting time + service time) of the jobs in getting phase I of service will be the setup time for first machine M_1 in phase II of service.

Step 1: Find the mean queue length on the lines of Gupta et al. (2011) using the formula

$$L = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} + \frac{\rho_4}{1-\rho_4} + \frac{\rho_5}{1-\rho_5} + \frac{\rho_6}{1-\rho_6}.$$

Step 2: Find the average waiting time of the customers on the line of Little's (1965) using relation $E(w) = \frac{L}{\lambda}$, where $\lambda = \lambda_1 + \lambda_2 + \lambda_1' + \lambda_2' + \lambda_3'$.

Step 3: Find the completion time(C) of jobs/customers coming out of Phase I .i.e. when processed through the network of queues (Phase I of service) by using the formula

$$C = E(W) + \frac{1}{\mu_1 p_{12} + \mu_1 p_{13} + \mu_2 p_{21} + \mu_2 p_{23} + \mu_3 + \mu_1' + \mu_2' + \mu_3'}.$$

Step 4: The completion time C of the customers / jobs through the network of queues (Phase I of service) will be the setup time for machine M_1 in Phase II of service. Define machines M_1, M_2, M_3 ------ M_m with processing time $A_{i1}' = A_{i1} + C$ and $A_{i,j}' = A_{i,j}$; $i = 1,2,3,...,n; j = 2,3,...,m$.

Step 5: Check the structural condition: $Min\ A_{i,s}' \geq Max\ A_{i,(s+1)}'$; s = 1, 2, 3, 4………

If these conditions are satisfied then go to Step 6, else the data is out of scope of present algorithm.

Step 6**:** Introduce two fictitious machines G and H with processing times G_i and H_i defined as

$G_i = A'_{i,1} + A'_{i,2} + A'_{i,3} + ... + A'_{i,(m-1)}$ and $H_i = A'_{i,2} + A'_{i,3} + A'_{i,4} + ... + A'_{i,m}$ for all i..

Step 7**:** Using Johnson's procedure, obtain the sequence {S} of jobs with minimum total elapsed time and calculate total elapsed time $t_{n,p}(S); p= 2,3,4...m$.

Step 8**:** Find the latest time L_m of m^{th} machine M_m for sequence {S} of jobs as $L_m(S) = t_{n,m}(S) - \sum_{i=1}^{n} A_{i,m}$.

Step 9**:** For the sequence {S} of jobs, calculate the latest time of machines M_j (j =2, 3, …., m-1) and their utilization time as follows:

$$Y_{1m}(S) = L_m(S) - A_{1,(m-1)}(S),\ Y_{qm}(S) = L_m(S) + \sum_{i=1}^{q-1} A_{i,m}(S) - \sum_{i=1}^{q} A_{1,(m-1)}(S); q = 2,3,......,n$$

$$L_{m-1}(S) = \min_{1\leq q\leq n}\{Y_{qm}(S)\},\ U_{m-1}(S) = t_{n(m-1)}(S) - L_{m-1}(S).$$

Now, for j= (m-1), (m-2)……..3; calculate

$$Y_{1j}(S) = L_j(S) - A_{1,(j-1)}(S),\ Y_{qj}(S) = L_j(S) + \sum_{i=1}^{q-1} A_{i,j}(S) - \sum_{i=1}^{q} A_{1,(j-1)}(S); q = 2,3,......,n$$

$$L_{(j-1)}(S) = \min_{1\leq q\leq n}\{Y_{qj}(S)\} \text{ and } U_{(j-1)}(S) = t_{n(j-1)}(S) - L_{(j-1)}(S).$$

Step 10**:** Find the total minimum rental cost of the machines for the optimal sequence {S} of jobs using the relation $R(S) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S) \times C_2 + U_3(S) \times C_3 + ---- + U_{m-1}(S) \times C_{m-1} + \sum_{i=1}^{n} A_{im} \times C_m$.

The sequence {S} will be the optimal sequence of jobs processing with minimum total elapsed time $t_{n,m}$, minimum rental cost of the equipments/machines R(S) under the restrictive hiring policy, with mean queue length L and average waiting time E(w).

5. TEST OF PROPOSED ALGORITHM

Consider fourteen customers / jobs are to be processed through the network of queues with the servers S_1, S_2 and S_3. Server S_3 is commonly linked in series with each of two servers S_1 and S_2. The number of the customers, mean arrival rate, mean service rate and associated probabilities are as given in table 1.

Table 1: The detail classification of the linkage model

S. No.	No. of Customers	Mean Arrival Rate	Mean Service Rate	Probabilities
1	$n_1 = 4$	$\lambda_1 = 3$	$\mu_1 = 10$	$p_{12} = 0.4$
2	$n_2 = 2$	$\lambda_2 = 5$	$\mu_2 = 9$	$p_{13} = 0.6$
3	$n_3 = 3$	$\lambda_1' = 2$	$\mu_1' = 7$	$p_{21} = 0.6$
4	$n_4 = 2$	$\lambda_2' = 4$	$\mu_2' = 8$	$p_{23} = 0.4$
5	$n_5 = 3$	$\lambda_3' = 5$	$\mu_3' = 9$	
6	$n_6 = 14$		$\mu_3 = 22$	

After getting Phase I of service, jobs/customers are to be processed on equipments/machines M_1, M_2, M_3 and M_4 with processing time $A_{i,1}$, $A_{i,2}$, $A_{i,3}$ and $A_{i,4}$ respectively as given in table 2.

Table 2: The machines M_1, M_2, M_3 and M_4 with processing times

Jobs	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$M_1(A_{i,1})$	15	20	18	17	16	19	17	20	15	16	17	19	18	15
$M_2\ (A_{i,2})$	14	13	12	14	10	11	12	13	11	12	13	14	10	12
$M_3\ (A_{i,3})$	9	8	9	10	7	8	10	9	7	8	10	7	8	9
$M_4\ (A_{i,4})$	6	5	3	5	4	3	6	5	4	5	3	2	6	4

The objective is to find an optimal sequence of the jobs / customers processing to minimize the makespan and rental cost of equipments/machines for the proposed queue-scheduling linkage model by considering the first phase service if the rental cost per unit of time for the machines M_1, M_2, M_3 and M_4 are 8, 10, 12 and 6 per unit of time.

Here, we observe that

$\rho_1 = 0.789474$, $\rho_2 = 0.96433$, $\rho_3 = 0.285714$, $\rho_4 = 0.5$, $\rho_5 = 0.555556$, $\rho_6 = 0.668262$.

The mean queue length = Average number of jobs / customers = L = 18.101925 units, and

Average waiting time of the jobs / customers =0.952733 units.

The total completion time of jobs/customers when processed through network of queues C = 0.96811 units.

On taking this completion time as the setup time for the first machine M_1 in Phase II of service and on using various steps of the proposed algorithm, we obtain S = 4 – 1 – 7 – 8 – 2 – 11 – 14 – 10 – 13 – 3 – 12 – 9 – 6 – 5 as an optimal sequence of jobs processing with minimum total elapsed time as 276.553619 units.

The latest times at which machine M_4, M_3 and M_2 should be taken on rent are L_4=215.553619 units, L_3=153.553619 units, and L_2= 94.553619 units.

The utilization time of machines M_2 and M_3 are $U_2(S)$ = 171, $U_3(S)$ = 119 units and the minimum possible rental cost of machines will be R(S) = 5200.428711 units.

6. CONCLUSION AND DISCUSSION

In this paper an algorithm is developed to minimize the hiring cost of equipments/machines taken on rent under a restrictive hiring policy for the proposed queuing scheduling linkage model. The latest times at which various equipments/machines must be hired so as to minimize the utilization time and hence their hiring/rental cost with minimum makespan, average waiting time and with minimum mean queue length are derived. Using Johnson's technique, the minimum makespan for the above discussed numerical illustration comes to be 276.553619 units with total possible hiring cost of equipments/machines taken on rent is 5538.5612 units. If the proposed algorithm is applied, the minimum makespan comes to be same and the total possible hiring cost of equipments/machines taken on rent under the same hiring policy come to be 5200.428711 units, which is comparatively less than the hiring cost as calculated by using Johnson's technique. Hence, the proposed algorithm is more efficient as it optimize both the criteria's simultaneously for a given phase I of the service. The study may further be extended by introducing the concept of fuzziness in processing of jobs/customers and by including some more constraints in the linkage model such as independent setup time, transportation time and weightage of jobs etc.

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