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Valuing flexible operating strategies in nickel production under uncertainty

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Abstract: With increasing global demand for nickel (which is a key component of stainless steel) the focus of mineral industry is currently on the abundant low-grade nickel laterite reserves. The extraction of nickel from the low-grade laterites is a technically difficult and expensive process and, as a result, the profitability of nickel production projects is highly affected by uncertainty over future market conditions. The project value can be increased by utilizing flexible operating strategies in response to changing future market conditions and Real Options analysis provides a suitable tool for optimizing flexible operating strategies over a long planning horizon in the face of uncertainty.

This paper presents the first study on the valuation of flexible operating strategies in a realistic nickel laterite production system under uncertainty of nickel price and exchange rate. In this paper, the production of ore from the three hypothetical nickel laterite mines being fed to a central processing facility is studied. The common features of nickel laterite production, such as a two-fraction (limonite and saprolite) structure of the laterite ore body, layering of each fraction, with different ore grades (concentration of nickel) in each layer, and a simultaneous mining of several ore bodies, are incorporated into the model.

It is common in the minerals industry that the ore from each mine is blended to provide as constant a feed rate and grade as possible to the processing facility. However, such a constant feed strategy may not generate the best financial return. In this paper, we investigate whether higher returns can be achieved by adopting a flexible strategy of switching, at prescribed intervals of time, between different feed rates of ore from the three mines that have different quality of nickel laterite. Such flexible strategy allows the operator to change the production rate of nickel in response to changing projected market conditions.

In this paper, we use an approximate stochastic dynamic programming framework in the form of the Least Squares Monte Carlo (LSM) method, which we extend to multiple switching options problem that incorporates complex features of nickel laterite production. In addition, an approach that combines a genetic algorithm (GA) with the Monte Carlo simulations is developed for preliminary assessment of options and for estimating the upper bounds on the strategy values.

We compare the value (in terms of the expected discounted cash flow) of the optimal profit-maximising switching strategy for 10 year planning horizon with the NPV value of a constant feed strategy, commonly used in the mining industry. Numerical results show that the flexibility to selectively blend the ore from each mine in response to projected market conditions considerably increases the expected cash flow and the probability of larger profits, while decreasing the probability of smaller profits.

Keywords: *Real options, stochastic dynamic programming, Least Squares Monte Carlo (LSM), nickel laterite, multiple mine-site operation, genetic algorithm*

1. INTRODUCTION

With increasing global demand for nickel, which is a key component of stainless steel, the abundant low-grade nickel laterite reserves are the focus of the mineral industry. The extraction of nickel from the low-grade laterites is a technically difficult and expensive process, and as a result, the profitability of nickel production project is highly affected by uncertainty over future market conditions. The project value can be increased by utilizing flexible operating strategies in response to changing projected market conditions, and Real Options analysis provides a suitable tool for optimizing flexible strategies in the face of uncertainty.

Valuation of strategic flexibility to expand, contract, defer or abandon production has been among the earliest and the most popular applications of real options analysis in resources industry (e.g., see a benchmark paper by Brennan and Schwartz (1995) and recent work by Tsekrekos et al (2012) and Ndiaye and Armstrong (2013)). New applications include, e.g., mine planning under uncertainty (Botin et al (2012) and Dimitrakopoulos and Abdel Sabour (2007)). Despite a long history of real options applications in minerals industry, the assumptions for these models are in general still overly simplistic. For example, a most common assumption is that the metal production rate is constant over time, which in most cases is far from reality (as discussed by Dimitrakopoulos and Abdel Sabour (2007)).

This paper presents the first study on the valuation of flexible operating strategies in a realistic nickel laterite production system under uncertainty of nickel price and exchange rate. This paper studies the production of ore from three hypothetical nickel laterite mines being fed to a central processing facility. The common features of nickel laterite production, such as a two-fraction (limonite and saprolite) structure of the laterite ore body, layering of each fraction, with different ore grades (concentration of nickel) in each layer, and a simultaneous mining of several ore bodies, are incorporated into the model. Conventionally, each mine's output is blended to provide as constant a feed rate and grade as possible to the processing facility (such strategy is known as a *constant feed* strategy). However, such strategy may not generate the best financial return. In this paper, we investigate the benefit of a flexible operating strategy of switching, at pre-determined decision times, between different feed rates of ore from three mines with different quality of ore. Such flexible strategy allows the operator to change the nickel production rate in response to changing market conditions, represented by future uncertain nickel price and exchange rate.

In this paper, we use a stochastic dynamic programming framework in the form of the Least Squares Monte Carlo (LSM) approach. The LSM approach, developed by Longstaff and Schwarz (2001) for pricing American options in financial engineering, has been increasingly used in real options analysis (see, e.g. Gamba (2009), Tsekrekos et al (2012)). In this paper, we extend the LSM to the problem of multiple switching between multiple options that incorporates complex features of nickel laterite production. We also suggest an approach that combines Genetic Algorithm with Monte Carlo simulations for preliminary assessment of options and for estimating the upper bounds on strategy values.

We compute and compare the value of the optimal profit maximizing flexible operating strategy with the NPV of the constant feed strategy, commonly used in practice, for a 10-year planning horizon. Numerical results show that the flexibility to selectively blend the ore from each mine in response to projected market conditions considerably increases the expected cash flow and the probability of larger profits, while decreasing the probability of smaller profits.

2. NICKEL LATERITE PRODUCTION PROCESS: COST MODEL

A nickel production system with the three hypothetical nickel laterite ore bodies that are mined simultaneously and fed to a central processing facility is shown in Fig. 1. Nickel laterite ore body consists of a limonite upper layer and a saprolite lower layer. The limonite fraction of the ore body has to be mined first to expose the saprolite fraction. Both the limonite and the saprolite fractions typically consist of sublayers with different ore grade (i.e., the concentration of nickel can be different) in each of them.

The cost model for the nickel laterite production is given in the Appendix. The rates of material and energy consumption are estimated through the life-cycle assessments of mining and mineral processing by Norgate and Jahanshahi, 2011. The costs and prices assumed for diesel, electricity and explosives are based on historical and forecast prices from a range of sources. Explosives' costs are assumed to be constant due to a lack of publically available historic data and labour costs have been assumed to be 15% of annual costs. A 50% premium on electricity prices is adopted to reflect the increased supply costs of operating in a remote area.

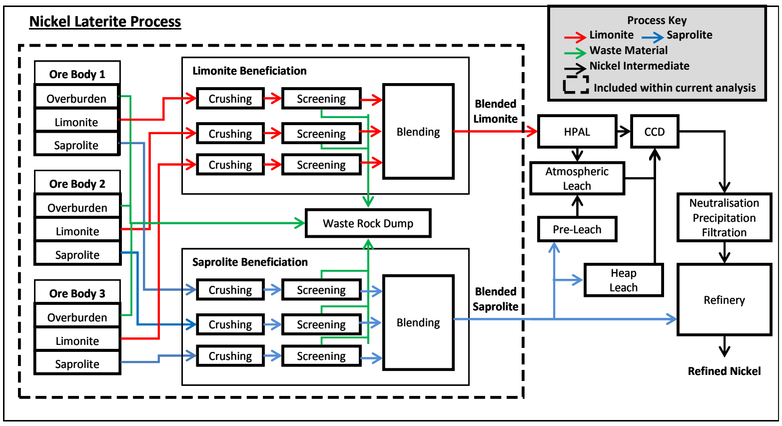

Figure 1: Schematics of nickel laterite production system.

The costs associated with switching production rates between different ore deposits (i.e. the movement of labour, trucks and equipment) have been assumed to be negligible when compared with the overall costs of a mining operation. Such assumption is justified as the ore production capacity from a mine usually exceeds the feed capacity of associated processing facilities. Operational flexibility is created by stockpiling ore material to allow for scheduled maintenance of mine equipment, and this can also coincide with the transport of mine equipment between ore deposits when changing operational strategies.

3. ORE FEED RATE OPTIONS MODEL

We consider the nickel price, $N(t)$, and the AUD/USD exchange rate, $X(t)$, as stochastic risk factors in this model. The reserve sizes for the limonite and saprolite fractions of the three ore bodies are given by the vector $Q(t) = (Q_1^l(t), Q_1^s(t), Q_2^l(t), Q_2^s(t), Q_3^l(t), Q_3^s(t))$, $\left(Q_i^l(t) \geq 0, Q_i^s(t) \geq 0\right)$, where t is the time. Each fraction consists of several layers of ore with different nickel grades. At the start of operations, the reserve sizes are given by $Q(0) = (Q_1^l(0), Q_1^s(0), Q_2^l(0), Q_2^s(0), Q_3^l(0), Q_3^s(0))$. The change in the reserve size is described by

$$dQ = -q(t)dt, \tag{1}$$

where q is the vector of the feed rates from the limonite and saprolite fractions of each ore body, $q(t) = (q_1^l(t), q_1^s(t), q_2^l(t), q_2^s(t), q_3^l(t), q_3^s(t))$. We assume that the operator can choose, at given intervals of time, the feed rates of the limonite from each ore body q_i^l, $i = 1, 2, 3$, (they are shown as red arrows prior to the beneficiation process in Fig. 1), so that the total feed rate of limonite ore q^l is constant over time, $q^l = q_1^l + q_2^l + q_3^l = const$. The feed rate of the saprolite is then computed based on the assumption that the limonite and the saprolite are mined in the same proportions of their respective reserve sizes. Thus, the feed rates of the limonite from each ore body (q_1^l, q_2^l, q_3^l) fully determine the operating strategy for a given time period and can be viewed as control (or decision) variables in this problem.

It is convenient to discretise the decision variables in the following way. We define the following set of non-dimensional feed rates of limonite: $(0, 0.25, 0.5, 0.75, 1)$. The discrete values for the feed rate of limonite are defined as all possible combinations of feed rates from each ore body, that satisfy $q_i^l = q^l \overline{q}_i$, $i = 1, 2, 3$, $\sum_{i=1}^{3} \overline{q}_i = 1$. This gives the set of 15 options $O = \{o^{(1)}, ..., o^{(15)}\}$:

$$\begin{aligned} &o^{(1)} = (0, 0, 1);\ o^{(2)} = (0, 0.25, 0.75);\ o^{(3)} = (0, 0.5, 0.5);\ o^{(4)} = (0, 0.75, 0.25);\ o^{(5)} = (0, 1, 0);\\ &o^{(6)} = (0.25, 0, 0.75);\ o^{(7)} = (0.25, 0.25, 0.5);\ o^{(8)} = (0.25, 0.5, 0.25);\ o^{(9)} = (0.25, 0.75, 0);\ o^{(10)} = (0.5, 0, 0.5);\\ &o^{(11)} = (0.5, 0.25, 0.25);\ o^{(12)} = (0.5, 0.5, 0);\ o^{(13)} = (0.75, 0, 0.25);\ o^{(14)} = (0.75, 0.25, 0);\ o^{(15)} = (1, 0, 0). \end{aligned} \tag{2}$$

We consider the time horizon T and assume that a decision to switch between the available options can be made on fixed dates $0 = t_1 < t_2 < ... < t_N = T$. The *cash flow* for the time interval $[t_{k-1}, t_k]$ is given by

$$\pi(N_k, X_k, Q_k, o_k, t_k) = g(Q_k, o_k)N_k - X_k q_k^{tot} P(Q_k, o_k), \tag{3}$$

where the subscript k denotes the values at time t_k, $g(Q_k, o_k)$ and $P(Q_k, o_k)$ are the nickel production rate and the unit production cost associated with the reserve state Q_k and the option o_k; q_k^{tot} is the total input rate of ore for a time interval of interest (both functions g and P are determined by the cost model described in the Appendix). The *objective* is to maximize the expected discounted cash flow over the time horizon.

We assume that the nickel price N follows a geometric Brownian motion (GBM) stochastic process

$$dN = \mu N dt + \sigma_{Ni} N dW_1, \tag{4}$$

where μ is the drift, σ_{Ni} is the volatility (constant), W_1 is the Wiener process. For the exchange rate X, we use the mean-reverting stochastic process in the form

$$d \ln X(t) = (\theta - a \ln X(t))dt + \sigma_X dW_2, \tag{5}$$

where W_1 and W_2 are independent Wiener processes; θ, a and σ_x are the constants.

4. METHODOLOGY

In this paper, we use a stochastic dynamic programming framework in the form of the Least Squares Monte Carlo (LSM) method, which we extend to multiple switching options problem that incorporates path-dependency and complex features of nickel laterite production. In addition, an approach that combines a genetic algorithm (GA) with the Monte Carlo simulations is used for preliminary assessment of options and for estimating the upper bounds on the strategy values.

4.1. Least Squares Monte Carlo (LSM) Approach

The approach in this paper is based on the work by Gamba (2009) and Tsekrekos et al (2012). The Bellman value function for the optimal switching problem described in Section 3 is given by

$$V(S_k, Q_k, o_k, t_k) = \max_{o \in O} \left\{ \pi(S_k, Q_k, o, t_k) + e^{-r\Delta t} E_k[V(S_{k+1}, Q_{k+1}, o_{k+1}, t_{k+1}) | S_k, Q_k, o] \right\} \tag{6}$$

where $S_k \equiv (N_k, X_k)$, r is the risk-free interest rate and $E_k[.]$ is the expectation (under the equivalent martingale measure) conditional on the information available at time t_k. We denote by $\Phi(S_k, Q_k, o, t_k) = e^{-r\Delta t} E_k[V(S_{k+1}, Q_{k+1}, o_{k+1}, t_{k+1}) | S_k, Q_k, o_k]$ the continuation function in the Bellman equation (6) and use the LSM approach to approximate the continuation function numerically. Following Longstaff and Shwartz, (2001), we approximate the continuation function by a finite set of basis functions $\{\phi_l, l = 0, ..., L\}$, as $\Phi(S_k, Q_k, o_k, t_k) \approx \sum_{l=0}^{L} \beta_l(Q_k, o_k, t_k)\phi_l(S_k)$. We define the finite set of possible reserves states $\mathbb{Q} = \{Q^j, j = 1, ..., J\}$ and generate the Monte Carlo scenarios $\{S_k^m, m = 1, ..., M\}$ according to Eqs.(4,5). The approximate continuation functions are computed by backward recursion as follows. For each time step $t_k, k = N-1, ..., 2,$ we

(1) compute the continuation functions for each option $o_k \in O$ and every reserve state $Q_k \in \mathbb{Q}$ via the least-squares fitting. The coefficients β_i are found as:

$$\{\hat{\beta}_l(Q_k, o_k, t_k)\}_{l=0}^{L} = \arg\min_{\{\beta_l\}_{l=0}^{L}} \sum_{m=1}^{M} \left\{ \left[\sum_{l=0}^{L} \beta_l(Q_k, o_k, t_k)\phi_l(S_k^m) - e^{-r(t_{k+1}-t_k)} G^m(t_{k+1}, t_N) \Big| S_k^m, Q_k, o_k \right]^2 \right\},$$

where $G^m(t_{k+1}, t_N) | S_k^m, Q_k, o_k$ is the optimal cash flow on $[t_{k+1}, t_N]$ for m-th Monte Carlo path, conditional on the information at time t_k;

(2) find optimal options by maximizing the Bellman value function:

$$o_k^* = \arg\max_{o_k \in O} \{\pi(S_k, Q_k, o_k, t_k) + \Phi(S_k, Q_k, o_k, t_k)\};$$

(3) compute the optimal cash flow $G^m(t_k, t_N)$ for each Monte Carlo path and state of reserve, that corresponds to optimal option o_k^*.

The computed continuation functions are then used to determine the optimal strategies for each Monte Carlo path. In order to reduce the number of possible states, we mapped the reserve states onto the nickel grades in the layers of the three ore bodies. We tested the monomial, Laguerre and Hermite basis functions using the total standard deviation of the least squares residuals as a measure of goodness-of-fit. All the basis functions provided very similar results, with Laguerre polynomials giving the best fit.

4.2. Genetic Algorithm

The genetic algorithm starts with the random initial population of chromosomes, consisting of genes representing a decision. In this study, each gene represents one of the 15 options in (2), and the number of genes in each chromosome is determined by the number of years in the planning horizon. Tournament selection, one-point crossover and bit-wise mutation are used in this study based on the code published at the Kanpur Genetic Algorithms Laboratory website (http://www.iitk.ac.in/kangal/codes.shtml). Other associated parameters of GA are set as: number of bits = 6, pc = 0.9, pm = 0.05, population size = 100, number of generations = 10000, number of runs = 5, tournament size=10 and the algorithm terminates if there is no improvement in the results after 100 iterations.

We define the value of the optimal strategy as an expected discounted cash flow over all Monte Carlo scenarios, conditional on the feed rate at t_1. The feed rates at $t_k, k = 2, ..., N-1$, are computed using the GA, while the feed rate at t_1 is selected so that it maximizes the expected discounted cash flow over all Monte Carlo scenarios.

5. NUMERICAL RESULTS AND DISCUSSION

In this section, we consider a 10-year planning horizon, with operating decisions taken annually. We compare the value of the constant feed strategy with the value of the long-term profit maximizing strategy.

A projected price of nickel from Mulshaw (2012) is used as the mean forecast of the nickel futures price, while the volatility is estimated from the historical data. We use the historical nickel commodity spot prices traded in US dollar from the World Bank from May 1983 to May 2013. In this case we have 361 end-of-month price observations and log-return rates. We then assume that the implied volatility of the nickel price for this period is constant and is approximated by the volatility of the historical spot price sample. For the sampled historical data, the calculated volatility is 31.94 % annually. For the exchange rate of AUD/USD currency pair, we use the implied volatility of mid bid-ask at-the-moneyness (ATM) quotations published by the Federal Reserve Bank of New York. The maturities available are one week and one-, two-, three-, six-, twelve-month and two-year. We take the average value of the implied volatility across all the seven maturities quoted on 31 May 2013 which is 11.61%. We use the historical 20 year average exchange rate as the future mean value for the exchange, because it is difficult to establish any consensus forecast and the AUD/USD exchange rate for such a long future period. The model parameters are estimated as in Zhu et al (2009). The commonly used in practice constant feed strategy blends the ore from each mine to provide as constant ore volume and nickel laterite feed as possible. For this case study, the following sequence of operating options defined by (2) provides a constant feed strategy for a 10-year planning horizon: $(2, 8, 8, 8, 5, 5, 5, 15, 15, 15)$.

Table 1: Comparison of strategy values

Strategy type	Value (AU$)	Std error (AU$)
Constant feed	NPV=1520.6 M	4.97 M
Optimal strategy	Extended NPV = 1630.16 M	7.17 M

For the purpose of this study, we use $r = 0.1$.

For the Least Squares Monte Carlo approach, we use 90000 Monte Carlo simulations to calibrate the continuation functions and 10000 additional Monte Carlo simulations to compute the value of the optimal strategy. The real options value of the strategy can be viewed as the "extended NPV" that includes the value of flexibility to switch between different feed rates of ore from the three mines in the face of uncertainty. The same sample of 10000 Monte Carlo simulations is used to compute the NPV of the constant feed strategy as the expected discounted cash flow over all Monte Carlo simulations. The results are shown in Table 1 and Fig.2. We can see that over 7% improvement in the expected discounted cash flow over the 10 years can be obtained by using the long-term optimal strategy. The upper bound on the value of the flexible switching strategy obtained using

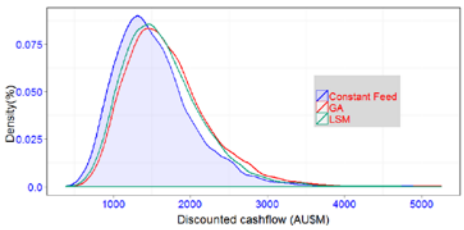

Figure 2: Discounted cash flow over 10000 Monte Carlo scenarios from different methods.

the GA algorithm is AU$1691.3M, which shows over 11% maximum possible improvement in the expected discounted cash flow over the 10 years as compared with the constant feed strategy. One can see from Fig.2 that the flexibility to selectively blend the ore from each mine in response to projected market conditions considerably increases the expected cash flow and the probability of larger profits, while decreasing the probability of smaller profits.

6. FUTURE DIRECTIONS

This paper assumes that the costs associated with switching production rates between different ore deposits (such as the movement of labour, trucks and equipment) are negligible due to the opportunities for stockpiling the ore material and transporting of mine equipment between ore deposits during scheduled maintenances. In reality, such opportunities may not exist because of the geographical and infrastructure constraints, inability to stockpile and inflexible work force, and therefore the costs of changing the production rates would need to be taken into account and might negatively affect the profit. Complexities around varying gangue, impurities and supply longetivity from different sources also affect the operating costs and the profit in real settings.

Further improvements in project profitability can potentially be achieved by incorporating additional flexibilities in the process, e.g., by allowing larger variations of throughput in response to future market uncertainty and also by considering the opportunity to switch between different processing routes to produce refined nickel (thus extending the study to the full production system as shown in Fig. 1). The real options model in this paper can be applied to the production processes of other metals, such as copper.

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APPENDIX: COST MODEL

Nickel production cost for the time interval $[t_{k-1}, t_k]$ is given by $P_k q_k^{tot} = CD_k + CE_k + CEX_k + CL_k$, where CP, CD, CE, CEX, CL are the diesel, electricity, explosives and labour cost at time t_k respectively. They are given by

$$CD_k = RD_k \sum_{i=1}^{3} D_{ik} q_{ik}, \quad CE_k = RE_k \sum_{i=1}^{3} (EMq_{ik} + EC(q_{ik}^l + q_{ik}^s)),$$
$$CEX_k = REX_k \sum_{i=1}^{3} EX(q_{ik}^l + q_{ik}^s + q_{ik}^o), \quad CL_k = 0.176\left(CD_k + CE_k + CEX_k\right),$$

where q_{ik} is the cumulative of limonite, q_{ik}^l, saprolite, q_{ik}^s, and overburden, q_{ik}^o, feed rates for the i-th ore body, RD_k is the unit price of producing energy using diesel, D_{ik} is the energy needed, RE_k is the unit electricity price, EM is the consumed electricity for mining and EC is the consumed electricity for crushing and gridding, REX_k is the unit explosives price, EX is the weight of explosives used per unit of ore. The unit prices for diesel, electricity and explosives are given in Table A1. The overburden feed rate q_{ik}^o is calculated as a fraction of limonite feed rate, $q_{ik}^o = q_{ik}^l f^o$, where f^o is 1.5, 1 and 3 for the first, second and third ore body respectively. The saprolite feed rate is computed as $q_{ik}^s = \left[q_{ik}^l Q_i^s(0)\right] / Q_i^l(0)$, where the sizes of the limonite and saprolite reserves $Q_i^l(0)$ and $Q_i^s(0)$ are given in Table A2.

Table A1: Prices of diesel, electricity and explosives

	YEAR	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Diesel	AU$/GJ	30.81	32.21	33.61	34.17	34.73	35.29	35.85	36.41	37.25	38.10
Electricity	AU$/kWh	0.075	0.083	0.090	0.093	0.096	0.099	0.102	0.105	0.108	0.111
Explosives	AU$/kg	0.555	0.555	0.555	0.555	0.555	0.555	0.555	0.555	0.555	0.555
	YEAR	2018	2019	2020	2021	2022	2023	2025	2026	2027	2028
Diesel	AU$/GJ	38.94	39.78	40.62	41.46	42.30	43.14	44.82	45.10	45.38	45.66
Electricity	AU$/kWh	0.114	0.117	0.120	0.123	0.126	0.129	0.135	0.138	0.141	0.144
Explosives	AU$/kg	0.555	0.555	0.555	0.555	0.555	0.555	0.555	0.555	0.555	0.555

Table A2: Reserve sizes

	Ore body 1	Ore body 2	Ore body 3
Limonite reserve (Mt)	60	14.6	5.2
Saprolite reserve (Mt)	40	50	10.95

The output nickel rate is calculated as follows:

$$g_k\left(Q_k, o\right) = \sum_{i=1}^{3}\left[grl_{ik} q_{ik}^l \left(1 - RejLim\right) + grs_{ik} q_{ik}^s \left(1 - RejSap\right)\right],$$

where grl_{ik} and grs_{ik} are the nickel grades in the current layer of the limonite and saprolite fractions respectively of the *i*-th ore body, *Rejlim* and *RejSap* are the percentage of rejected nickel in the limonite and saprolite screening process respectively (they are taken as 0.1 of total nickel produced). The sizes of the layers and the nickel grades in each layer are shown in Table A3 for the three ore bodies. The layers are mined consecutively, until the total reserve is completely exhausted.

Table A3: Layers with different grade of nickel

Ore body 1, limonite (Mt)	60-50	50-40	40-30	30-20	20-10	10-0
Nickel grade	0.0154	0.014	0.014	0.014	0.014	0.0126
Ore body 2, limonite (Mt)	14.6-6.8	6.8-0				
Nickel grade	0.016	0.014				
Ore body 3, limonite (Mt)	5.2-5	5-3.2	3.2-0			
Nickel grade	0.0114	0.008067	0.005822			
Ore body 1, saprolite (Mt)	40-30	30-20	20-10	10-0		
Nickel grade	0.025	0.024	0.024	0.023		
Ore body 2, saprolite (Mt)	50-40	40-30	30-20	20-10	10-0	
Nickel grade	0.009943	0.006214	0.004972	0.004972	0.004972	
Ore body 3, saprolite (Mt)	10.95-8.6	8.6-3.8	3.8-0			
Nickel grade	0.0118	0.009244	0.007437			