

On adoption of new technology under uncertainty

V. Gaitsgory ^a, T. Tarnopolskaya ^b

^a*School of Computer Science, Engineering and Mathematics, Flinders University, South Australia, Australia*

^b*CSIRO Computational Informatics, Sydney, Australia*

Email: Tanya.Tarnopolskaya@csiro.au

Abstract: Investment in new or improved technology is among the most important decisions that companies make, because of the initial cost associated with technology adoption and the impact on company's performance over many years. Company's decision about adoption of new technologies is a trade-off between the cost of making a mistake by adopting too soon and the opportunity cost of waiting for arrival of even better technology. The uncertainty in the speed of new technology arrivals and the extent of technological improvements influences the adoption decision.

This paper continues the line of research that considers the innovation process as a stochastic process with the improvements in new technology described by the Poisson jump process. The focus of early research was on innovation process characterised by a single stochastic variable (namely, the technological efficiency parameter) describing the extent of technology improvements. A critical (threshold) value of the efficiency that triggers technology adoption was established for such a case.

This paper studies the situation common in the minerals processing industry, where not only the efficiency of new technology under development, but also its operating costs may change in a random fashion. This paper extends previous research to the following situations: (1) the efficiency of new technology remains unchanged, while the operating costs decrease randomly, following the Poisson jump process; (2) both the efficiency and the operating costs of new technology change in a random fashion. This case studies two possibilities: (a) the operating cost is a function of the efficiency, and (b) both the efficiency and the operating costs of new technology follow the Poisson jump process with independent jump sizes but the same arrival times.

This paper establishes, for the first time, a threshold curve that separates the plane of feasible values of the efficiency and the operating costs of new technologies into two regions: (1) a waiting region, where new technology adoption is still not optimal and (2) an adoption region. The threshold curve represents a decision boundary that can assist companies in making optimal strategic decisions under uncertainty. Numerical illustrations of the behaviour of the threshold curve with change in model parameters describing the market conditions and the characteristics of the stochastic innovation process are provided. The results show that the adoption decision is significantly affected by the market price of the product (commodity), and the extent of technological improvements the company expects to occur over time.

Keywords: *Technology adoption, technological uncertainty, optimal timing, Poisson technology improvement process, threshold curve*

1 INTRODUCTION

Investment in new or improved technology is among the most important decisions that companies make, because of the initial cost associated with technology adoption and the impact on company's performance over many years. Empirical observations show that a substantial time lag typically exists between the arrival of new technology and its adoption (Doraszelski (2004)). Company's decision about adoption of new technologies is a trade-off between the cost of making a mistake by adopting too soon and the opportunity cost of waiting for arrival of even better technology. The uncertainty in the speed of new technology arrivals and the extent of technological improvements influences company's decision about new technology adoption.

A significant amount of literature is devoted to optimal technology adoption. The research on the topic can be broadly divided into two classes: (1) decision theoretic models and (2) game theoretic model. In the first class, the profit of the company is only influenced by its own technology adoption decision (see, e.g., Farzin et al. (1998), Gryglewicz et al. (2008), Doraszelski (2004)), while in the second class, the profit is influenced by the competition in the market (see, e.g., Huisman and Kort (2003) and the references therein). This paper follows the decision theoretic approach.

This paper continues the study by Farzin et al. (1998) and Huisman (2001) that considers the innovation process as a stochastic process with the technology improvement described by the Poisson jump process. The focus of early research (Farzin et al. (1998), Huisman (2001)) has been on the innovation process characterised by a single stochastic variable (namely, the technological efficiency parameter), describing the extent of technology improvements. A critical (threshold) value for the efficiency of new technology that triggers optimal technology adoption has been established for such a case.

We study the situation common in the minerals processing industry, where new processing technologies may differ from existing technologies in both the efficiency and the operating costs (see, e.g., Robinson and Treadgold (2013), Bao et al. (2013)). This paper extends previous research by Farzin et al. (1998) and Huisman (2001) to such situations, assuming that both the efficiency and the operating costs of new technology under development vary in a random fashion. We study the following situations: (1) the efficiency of new technology remains unchanged, but the operating costs decrease randomly, following the Poisson jump process; (2) both the efficiency and the operating costs of new technology change in random fashion. This case studies two possibilities: (a) the operating cost is a function of efficiency, and (b) both the efficiency and the operating costs of new technology follow the Poisson jump diffusion process with independent jump sizes but the same arrival times. The critical (threshold) values of the efficiency and the operating costs that trigger optimal technology adoption are established for these cases.

This paper establishes for the first time (to the best of the authors knowledge) a threshold curve that represents a locus of critical values for the efficiency and the operating cost of new technology. The threshold curve separates the plane of feasible values of the efficiency and the operating cost into two regions: (1) a waiting region, where new technology adoption is still not optimal and (2) an adoption region, where new technology adoption occurs. The threshold curve represents a decision boundary for optimal adoption of new technology under uncertainty. Numerical illustrations of the threshold curve and its behaviour with change in the model parameters are presented.

2 MODEL

We consider a risk-neutral company whose profit is only determined by its own technology choice. The technology used by the company is characterised by two parameters: the technological efficiency parameter $\zeta > 0$ and the unit operating cost $\omega > 0$. We study a dynamic model with infinite time horizon.

Company's production function is given by (see Farzin et al. (1998) and Huisman (2001))

$$h(v, \zeta) = \zeta v^\alpha, \tag{1}$$

where $v(\geq 0)$ is a variable input, $\alpha \in (0, 1)$ is the constant output elasticity. The profit flow is given by

$$\pi(\zeta, \omega) = \max_v (p\zeta v^\alpha - \omega v), \tag{2}$$

where $p(> 0)$ is the unit output price. The profit flow (2) can be re-written as

$$\pi(\zeta, \omega) = C_1 \zeta^{1/(1-\alpha)} \omega^{(1-\alpha)/\alpha}, \tag{3}$$

where $C_1 = (1 - \alpha)[\alpha^\alpha p]^{1/(1-\alpha)}$. If the company produces with the technology (ζ, ω) forever, its value is given by

$$V(\zeta, \omega) = \int_{t=0}^{\infty} \pi(\zeta, \omega) e^{-rt} dt = \pi(\zeta, \omega)/r. \quad (4)$$

We now assume that at the beginning of the planning horizon, the company produces with a technology whose efficiency equals ζ_0 and whose operations cost equals ω_0 . As time passes, new technologies that are characterised by new values of efficiency and operating costs appear (we assume that the company cannot influence the innovation process). We denote the efficiency of new technology by θ and the operating costs of new technology by η . We assume that the focus of the technological development is to either improve the efficiency, or to decrease the operating costs. When the efficiency of new technology is improved, the associated operating cost may be either higher or lower. It is also plausible that new technology would have decreased operating costs, but unaltered efficiency. Such cases will be studied in the following Section.

Other simplifying assumptions follow previous study by Farzin et al. (1998) and Huisman (2001). Thus, we assume that the company can adopt new technology only once by paying a sunk cost I .

3 THRESHOLD SWITCHING LEVELS

In this section we study several cases of the technological process development:

- The operating costs of new technologies follow a Poisson jump process, while the efficiency of new technologies remains unchanged;
- The efficiency of new technology follows a Poisson jump process, while the operating costs change as a function of efficiency.
- Both the efficiency and the operating costs follow a simultaneous synchronised changes according to the Poisson jump process.

3.1 Stochastic jump in the operating costs of new technologies

We assume that the efficiency of new technology remains unchanged, while the operating costs undergo random changes. For new technology to have a chance to be adopted, the operating cost should reduce. We therefore assume that the operating cost follows the Poisson process in the form:

$$d\eta = \begin{cases} v & \text{with probability } \lambda dt \\ 0 & \text{with probability } (1 - \lambda dt) \end{cases} \quad (5)$$

where v is a random variable uniformly distributed on the interval $[-\bar{v}, 0]$.

The profit flow function (3) for this case takes the form

$$\pi(\eta) = C_2(\eta)^{(1-\alpha)/\alpha}, \quad (6)$$

where $C_2 = (1 - \alpha)[\alpha^\alpha p]^{1/(1-\alpha)} \zeta^{1/(1-\alpha)}$.

We now make the following assumption.

Assumption 1. $\exists \eta^*$ such that if $\eta \leq \eta^*$ then it is optimal to adopt new technology, i.e., for $\forall \eta \leq \eta^*$, and for $\forall \omega \geq 0$, the value of the company $F(\eta, \omega)$ is given by

$$F(\eta, \omega) = V(\eta) - I, \quad (7)$$

where

$$V(\eta) = \int_{t=0}^{\infty} \pi(\eta) e^{-rt} dt = \frac{\pi(\eta)}{r}. \quad (8)$$

Given $(\eta, \omega) = (\eta_0, \omega_0)$, the Bellman equation can be written in the form

$$\begin{aligned} F(\eta_0, \omega_0) &= \int_0^{\Delta t} \pi(\omega_0) e^{-rt} dt + e^{-r\Delta t} E[F(\eta(\Delta t), \omega(\Delta t))] \\ &= \pi(\omega_0) \Delta t + (1 - r\Delta t) E[F(\eta(\Delta t), \omega(\Delta t))] + o(\Delta t), \end{aligned} \quad (9)$$

We now divide the interval $[-\bar{v}, 0]$ into two intervals: (1) $\{v : \eta + v \leq \eta^*\}$ and (2) $\{v : \eta + v > \eta^*\}$. Switching to new technology occurs in the first region and does not occur in the second region. Then the expectation $E[F(\eta(\Delta t), \omega(\Delta t))]$ can be written as

$$E[F(\eta(\Delta t), \omega(\Delta t))] = (1 - \lambda\Delta t)F(\eta_0, \omega_0) + \frac{\lambda\Delta t}{\bar{v}} \left[\int_{\eta^* - \eta_0}^0 F(\eta_0 + v, \omega_0) dv + \int_{-\bar{v}}^{\eta^* - \eta_0} F(\eta_0 + v, \omega_0 + v) dv \right]. \quad (10)$$

Substituting (10) into (9) yields, at the threshold ($\eta_0 = \eta^*$),

$$F(\eta^*, \omega_0) = \frac{\pi(\omega_0)}{r + \lambda} + \frac{\lambda}{(r + \lambda)\bar{v}} \int_{-\bar{v}}^0 (V(\eta^* + v) - I) dv \quad (11)$$

Using the continuity condition for the value function F between the region just prior to technology adoption and after the adoption, and also substituting $V(\cdot)$ from (8) into (11) and integrating yields

$$\frac{\pi(\omega_0)}{r + \lambda} + I \frac{r}{r + \lambda} + \frac{C_2 \lambda \alpha}{(r + \lambda) \bar{v} r} [(\eta^*)^{(1/\alpha)} - (\eta^* - \bar{v})^{1/\alpha}] = C_2 \frac{(\eta^*)^{(1-\alpha)/\alpha}}{r}. \quad (12)$$

Expression (12) represents a nonlinear equation in η^* that establishes the critical (threshold) value of the operating cost that triggers the technology adoption.

3.2 Stochastic jumps in the efficiency and operating costs

Operating cost is a function of efficiency. In this section we assume that new technology has a jump-improvement in the efficiency accompanied by the change in the unit operating cost. We denote by ζ_0 and ω_0 the efficiency and the operating costs of the existing technology used by the company. The efficiency and the operating costs of new technology under development are θ_0 and η_0 . The unit operating cost of new technology is assumed to be a function of efficiency of the form

$$\eta(\theta) = \eta_0(1 + \beta(\theta - \theta_0))^\gamma, \quad (13)$$

where $\beta > 0$ and $\gamma > 0$ are the constant parameters. We assume that the efficiency of new technology follows the Poisson jump process

$$d\theta = \begin{cases} u & \text{with probability } \lambda dt, \\ 0 & \text{with probability } (1 - \lambda dt), \end{cases} \quad (14)$$

where u is a random variable uniformly distributed on the interval $[0, \bar{u}]$.

The profit flow of new technology in this case is given by

$$\pi(\theta) = C_3 \theta^{1/(1-\alpha)} [(1 + \beta(\theta - \theta_0))^\gamma]^{(1-\alpha)/\alpha}, \quad (15)$$

where $C_3 = (1 - \alpha)[(\alpha/\eta_0)^\alpha p]^{1/(1-\alpha)}$.

While the efficiency of the new technology increases with each new technology arrival, the operating costs may either decrease or increase, depending on the parameters β and γ in (13). Clearly, it can only be optimal to switch to new technology if the profit function is an increasing function in θ .

Assumption 2. $\pi(\theta)$ is an increasing function of θ .

Assumption 3. $\exists \theta^*$ such that if $\theta \geq \theta^*$ then it is optimal to adopt new technology, i.e., for $\forall \theta \geq \theta^*$, and for $\forall \zeta \geq 0$, the value of the company $F(\theta, \zeta)$ is given by

$$F(\theta, \zeta) = V(\theta) - I, \quad (16)$$

where

$$V(\theta) = \int_{t=0}^{\infty} \pi(\theta) e^{-rt} dt = \frac{\pi(\theta)}{r}. \quad (17)$$

Given $(\theta, \zeta) = (\theta_0, \zeta_0)$, the Bellman equation can be written in the form

$$\begin{aligned} F(\theta_0, \zeta_0) &= \int_0^{\Delta t} \pi(\zeta_0) e^{-rt} dt + e^{-r\Delta t} E[F(\theta(\Delta t), \zeta(\Delta t))] \\ &= \pi(\zeta_0) \Delta t + (1 - r\Delta t) E[F(\theta(\Delta t), \zeta(\Delta t))] + o(\Delta t). \end{aligned} \quad (18)$$

We now divide the interval $[0, \bar{u}]$ into two intervals: (1) $\{u : \theta_0 + u \geq \theta^*\}$ and (2) $\{u : \theta_0 + u < \theta^*\}$. Switching to new technology occurs in the first region and does not occur in the second region. Then, the expectation $E[F(\theta(\Delta t), \zeta(\Delta t))]$ can be written as

$$\begin{aligned} E[F(\theta(\Delta t), \zeta(\Delta t))] &= (1 - \lambda\Delta t) F(\theta_0, \zeta_0) + \frac{\lambda\Delta t}{\bar{u}} \left[\int_0^{\theta^* - \theta_0} F(\theta_0 + u, \zeta_0) du \right. \\ &\quad \left. + \int_{\theta^* - \theta_0}^{\bar{u}} F(\theta_0 + u, \zeta_0 + u) du \right]. \end{aligned} \quad (19)$$

Substituting (19) into (18) yields, at the threshold $(\theta_0 = \theta^*)$,

$$F(\theta^*, \zeta_0) = \frac{\pi(\zeta_0)}{r + \lambda} + \frac{\lambda}{(r + \lambda)\bar{u}} \int_0^{\bar{u}} (V(\theta^* + u) - I) du \quad (20)$$

Using the matching (continuity) condition for the value function F between the region just prior to the adoption and after the adoption of new technology and rearranging gives the following nonlinear integral equation in θ^* that establishes the threshold value of the technology efficiency that triggers new technology adoption:

$$\frac{\pi(\zeta_0)}{r + \lambda} + I \frac{r}{r + \lambda} + \frac{\lambda}{r(r + \lambda)\bar{u}} \int_0^{\bar{u}} \pi(\theta^* + u) du = \frac{\pi(\theta^*)}{r}. \quad (21)$$

Independent synchronized jumps in the efficiency and the operating cost. We now assume that both the efficiency of new technology θ and its operating cost η follow Poisson processes (5) and (14) with common arrival times T_i .

Assumption 4. $\exists \theta^*$ and η^* such that if $\theta \geq \theta^*$ and $\eta \leq \eta^*$ then it is optimal to adopt new technology, that is, for $\forall \theta \geq \theta^*$, $\eta \leq \eta^*$, and for $\forall \zeta, \omega \geq 0$,

$$F(\theta, \eta, \zeta, \omega) = V(\theta, \eta) - I, \quad (22)$$

where

$$V(\theta, \eta) = \int_{t=0}^{\infty} \pi(\theta, \eta) e^{-rt} dt = \frac{\pi(\theta, \eta)}{r}. \quad (23)$$

where $\pi(\theta, \eta)$ is given by (3).

Given $(\theta, \eta, \zeta, \omega) = (\theta_0, \eta_0, \zeta_0, \omega_0)$, the Bellman equation can be written in the form

$$\begin{aligned} F(\theta_0, \eta_0, \zeta_0, \omega_0) &= \int_0^{\Delta t} \pi(\zeta_0, \omega_0) e^{rt} dt + e^{-r\Delta t} E[F(\theta(\Delta t), \eta(\Delta t), \zeta(\Delta t), \omega(\Delta t))] \\ &= \pi(\zeta_0, \omega_0) \Delta t + (1 - r\Delta t) E[F(\theta(\Delta t), \eta(\Delta t), \zeta(\Delta t), \omega(\Delta t))] + o(\Delta t). \end{aligned} \quad (24)$$

We now divide the rectangular region $[0, \bar{u}] \times [-\bar{v}, 0]$ into four regions: (1) $\{u, v : \theta_0 + u \geq \theta^*, \eta_0 + v \leq \eta^*\}$; (2) $\{u, v : \theta_0 + u \geq \theta^*, \eta_0 + v > \eta^*\}$; (3) $\{u, v : \theta_0 + u < \theta^*, \eta_0 + v \leq \eta^*\}$ and (4) $\{u, v : \theta_0 + u < \theta^*, \eta_0 + v > \eta^*\}$. Switching to new technology occurs in the first region, and does not occur in the second,

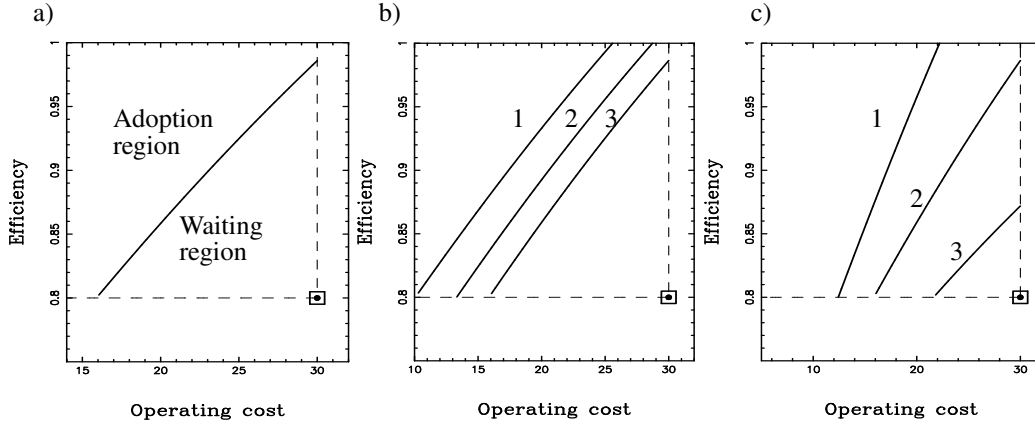


Figure 1. Threshold curve; (a) threshold curve separates the *waiting region* where adoption of new technology is still not optimal, from the *adoption region* where new technology adoption occurs. A dot in a square shows the initial values of the operating costs and the efficiency of the technology in use; (b) Effect of the product/commodity price p on the threshold curve behaviour, 1: $p = 10$, 2: $p = 15$, 3: $p = 100$; (c) Effect of the expected maximum size of efficiency improvement \bar{u} on the threshold curve behaviour, 1: $\bar{u} = 0.25$, 2: $\bar{u} = 0.2$, 3: $\bar{u} = 0.16$.

third and fourth regions. The expectation $E[F(\theta(\Delta t), \eta(\Delta t), \zeta(\Delta t), \omega(\Delta t))]$ in (24) can be written as

$$\begin{aligned}
E[F(\theta(\Delta t), \eta(\Delta t), \zeta(\Delta t), \omega(\Delta t))] &= (1 - \lambda \Delta t) F(\theta_0, \eta_0, \zeta_0, \omega_0) \\
&+ \frac{\lambda \Delta t}{\bar{u} \bar{v}} \left\{ \int_{\theta^* - \theta_0}^{\bar{u}} \int_{-\bar{v}}^{\eta^* - \eta_0} F(\theta_0 + u, \eta_0 + v, \zeta_0 + u, \omega_0 + v) dudv \right. \\
&+ \int_0^{\theta^* - \theta_0} \int_{-\bar{v}}^{\eta^* - \eta_0} F(\theta_0 + u, \eta_0 + v, \zeta_0, \omega_0) dudv \\
&\left. + \int_{\theta^* - \theta_0}^{\bar{u}} \int_{\eta^* - \eta_0}^0 F(\theta_0 + u, \eta_0 + v, \zeta_0, \omega_0) dudv + \int_0^{\theta^* - \theta_0} \int_{\eta^* - \eta_0}^0 F(\theta_0 + u, \eta_0 + v, \zeta_0, \omega_0) dudv \right\} \quad (25)
\end{aligned}$$

At the threshold, $(\theta_0 = \theta^*, \eta_0 = \eta^*)$, (25) takes the form

$$\begin{aligned}
E[F(\theta(\Delta t), \eta(\Delta t), \zeta(\Delta t), \omega(\Delta t))] &= (1 - \lambda \Delta t) F(\theta^*, \eta^*, \zeta_0, \omega_0) \\
&+ \frac{\lambda \Delta t}{\bar{u} \bar{v}} \int_0^{\bar{u}} \int_{-\bar{v}}^0 (V(\theta^* + u, \eta^* + v) - I) dudv. \quad (26)
\end{aligned}$$

Substituting (26) into (24) yields, using the matching (continuity) condition for the value function F between the region just prior to technology adoption and after the adoption, yields

$$\frac{\pi(\zeta_0, \omega_0)}{r + \lambda} + I \frac{r}{(r + \lambda)} + \frac{\lambda(1 - \alpha)(\alpha p)^{1/(1 - \alpha)}}{(r + \lambda)r\gamma\bar{u}\bar{v}} [(\theta^* + \bar{u})^\gamma - (\theta^*)^\gamma][(\eta^*)^{1/\alpha} - (\eta^* - \bar{v})^{1/\alpha}] = \frac{\pi(\theta^*, \eta^*)}{r}, \quad (27)$$

where $\gamma = (2 - \alpha)/(1 - \alpha)$. Expression (27) describes the locus of the critical values of the efficiency θ^* and the operating costs η^* , which we call a *threshold curve*, and that triggers new technology adoption once it has been crossed.

Remark. Note that in the limit $\bar{v} \rightarrow 0$, expression (27) reduces to the expression for the critical efficiency level by Farzin et al. (1998). In the limit $\bar{u} \rightarrow 0$, expression (27) reduces to the expression for the critical operating cost level (12) derived in this paper.

4 NUMERICAL EXAMPLE

In this section, we study the case when both the efficiency of new technology θ and its operating cost η follow the Poisson process with common arrival times, and compute the threshold curve presented by (27). In this

example, the following model parameters are used: $\alpha = 0.5, s = 200.0, \zeta_0 = 0.8, \omega_0 = 30.0, \lambda = 1.0, \bar{u} = 0.2, r = 0.1, I = 1600.0$. In addition, we take $\bar{v} = 9.0$. Figure 1a shows the threshold curve ($\theta^* = \theta^*(\eta^*)$) computed from (27) under the conditions $\eta^* \leq \omega_0, \theta^* \geq \zeta_0$. The threshold curve separates the plane of feasible values of the efficiency and the operating costs of new technologies into two regions: (1) a *waiting region*, where new technology adoption is still not optimal and (2) an *adoption region* where adoption of new technology occurs.

The behaviour of the threshold curve with change in the model parameters is illustrated in Figs.1b and 1c. One can see from Fig.1b that increase in market price of the product p leads to faster technology adoption. On the other hand, Fig. 1c shows that increase in the expected maximum size of efficiency improvements \bar{u} of new technology delays the adoption of new technology.

5 DISCUSSION AND FUTURE WORK

This paper establishes the critical (threshold) values and the threshold curve for new technology adoption for the cases when the innovation process is characterised by two stochastic variables: the efficiency and the operating cost. The threshold values and the threshold curve depend on the model parameters that reflect the market conditions, the company's initial technological attributes and the characteristics of the stochastic innovation process.

The threshold curve established in this paper represents the boundary between the two decision regions (waiting and adoption) and may be a useful tool for decision making about optimal adoption of new technology under technological uncertainty. Numerical results show that an increase in the market price of the product leads to faster technology adoption, while an increase in the expected maximum size of efficiency improvements of new technology delays the adoption of new technology.

The expression for the threshold curve in this paper is derived for the case when the efficiency of new technology grows randomly, while the operating costs randomly decrease. In practice, it is not uncommon that an increase in the efficiency of new technology may be accompanied by an increase in the operating costs. Such case will be a subject of further study. An extension of the results in this paper to the uncertain market conditions also requires further study.

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