

Computer modelling and simulation of the mechanical response of composite lattice structures

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Abstract: Composite lattice shells are highly efficient and extensively used in various structural applications, such as rocket interstages, payload adapters for spacecraft launchers, fuselage components for aerial vehicles, and components of the deployable space antennas. The aim of this paper is to present an analytical approach based on the finite element discrete modelling that is capable of predicting mechanical behaviour of composite lattice structures with sufficient accuracy and, at the same time, is affordable in terms of computational expenses. This allows the modelling approach proposed in this work to enable the efficient solution of relevant design and design optimisation problems.

Structural analyses reported in the literature have been conventionally performed for composite cylindrical lattice shells. The paper investigates buckling behaviour of anisogrid composite lattice cylindrical, conical, and parabolic shells. The lattice shells are modelled as three-dimensional frame structures composed of curvilinear ribs subjected to the tension/compression, bending in two planes and torsion. The specialised finite-element model generation procedure (model generator/design modeller) is developed to control the orientation of the beam elements allowing the original twisted geometry of the curvilinear ribs to be closely approximated.

The generation of discrete models are presented and explained in detail for cylindrical and conical lattice shells. Buckling analyses are performed for the cylindrical and conical shells subjected to axial compressive loading, and for the structure composed of parabolic lattice shell loaded by transverse concentrated external load.

The effects of varying the length of the shells, the number of helical ribs and the angles of their orientation on the buckling behaviour of lattice structures are investigated. Critical buckling loads and corresponding buckling mode shapes are determined based on the modelling approach proposed in this work. The effects of parameters of the lattice structure on the values of critical buckling loads, mode shapes are examined using parametric analyses. Based on the computations, the angles of orientation of helical ribs delivering maximum critical loads for a number of particular structural designs are identified.

The results of these studies indicate that the modelling approach presented in this work can be successfully applied to the solution of design problems formulated for composite lattice shells.

Keywords: *Composite lattice shells, finite-element modelling, spacecraft structures, buckling analysis, critical buckling loads*

1. INTRODUCTION

Composite lattice cylindrical and conical shells made of unidirectional high-modulus carbon fibre reinforced plastics by continuous filament winding are increasingly used as structural components of spacecraft. Examples of such applications can be found in the designs of lattice anisogrid bodies of spacecraft and spokes of transformable/deployable umbrella like space antennas see e.g. Vasiliev et al. (2012). A typical design of a spacecraft with its body made in the form of a lattice cylindrical shell attached to a conical lattice payload adapter is shown in Figure 1. The lattice shells shown in the figure should provide the necessary level of mechanical structural performance during the launch and injection into orbit phases when the high intensity loadings are exerted on these structures. They normally include axial compression, transverse and pure bending, and torsion. Buckling under these loadings is one of the typical modes of failure of these shells.

The discrete models are normally built using finite-element modelling and could be composed of the beam, shell, or solid elements. Results of finite-element analyses of composite lattice shells can be found in the papers published by Fan et al. (2009), Frulloni et al. (2007), Hou and Gramoll (1998), Morozov et al. (2011a,b), and Zhang et al. (2009).

In this paper, an approach based on the finite element discrete modelling that is capable of simulating mechanical behaviour of composite lattice structures with sufficient accuracy and is affordable in terms of computational expenses is discussed.

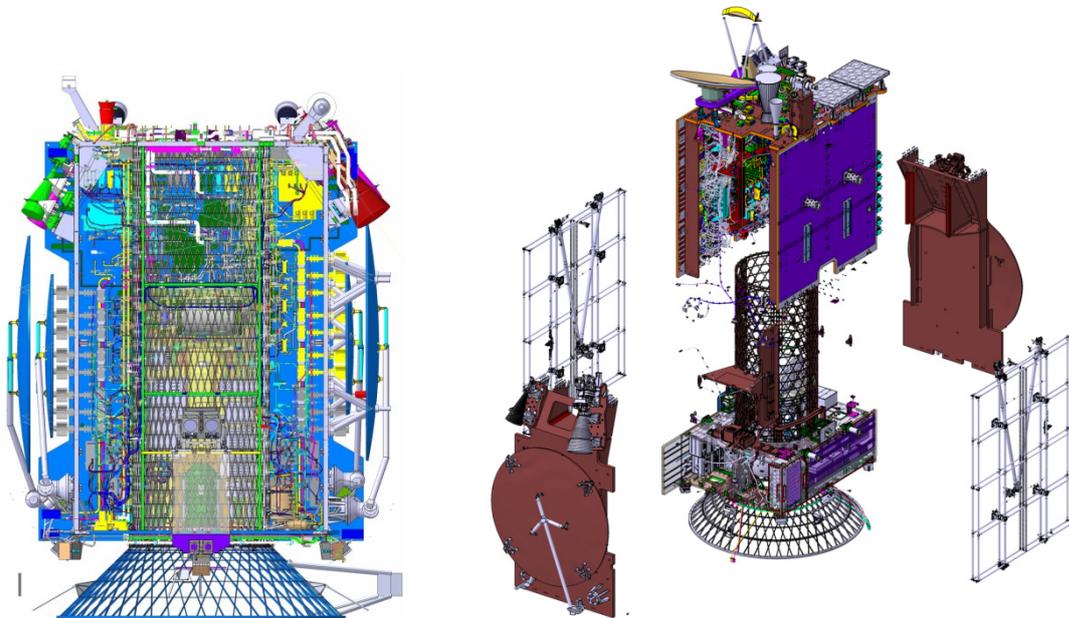


Figure 1. Spacecraft assembly including the composite lattice cylindrical body attached to the composite conical lattice adapter (Courtesy of ISS-Reshetnev Company).

2. FINITE-ELEMENT MODELLING AND SIMULATION

The lattice shells are modelled in this work as three-dimensional frame structures composed of the curvilinear ribs subjected to tension-compression, bending in two planes, and twisting. Using internal coding facilities of finite-element software packages, a model generator program can be developed to enable the geometry and finite-element model of the lattice shell to be automatically created. The generator/design modeller is capable of modelling the original geometry of the naturally twisted shapes of the helical ribs composing the lattice structure with maximum accuracy and authenticity. An additional advantage is that the size of the generated finite element model does not impede the efficiency of the computational process even for the lattice shells having a large number of ribs.

2.1. Cylindrical lattice shells

A composite lattice cylindrical shell is normally composed of two systems of filament wound ribs: a symmetric system of helical ribs and a system of circumferential ribs. The helical ribs make angles $\pm\phi$ with

the cylinder's generator line. The circumferential (hoop) ribs are passing through the middle points of the segments of helical ribs situated between the intersections as shown in Figure 2a. This arrangement of the ribs is referred to as the hexagonal lattice pattern. The geometry modelling of the lattice shell structure is based on four input variables, i.e. the overall length and diameter, L and D , the helical angle, ϕ , and the number of ribs, n , representing the system that has only one helical direction (either $+\phi$, or $-\phi$). The model is generated using a single typical unit cell shown in Figure 2a.

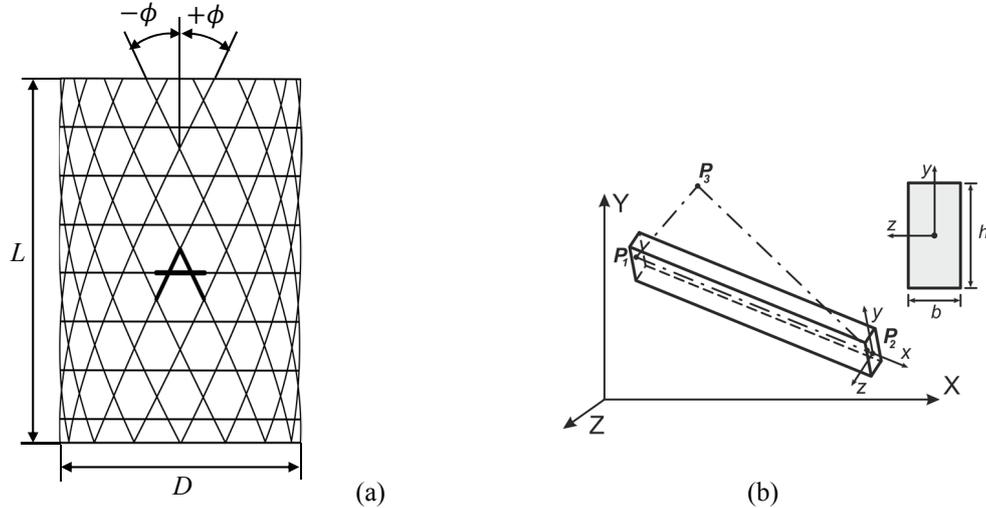


Figure 2. Lattice cylindrical shell (a) and linear finite beam element (b).

The lattice shell is modelled as a three-dimensional frame structure. The linear finite element having a $b \times h$ rectangular cross section and referred to the local coordinate frame xyz (see Figure 2b) is employed to model the ribs. Each of the element nodes, P_1 and P_2 is characterised by six degrees of freedom, i.e. by three displacements and three rotations. The key point P_3 , situated in the local plane xy , is used to orient the element with regard to the global frame XYZ as shown in Figure 2b. Using this element, the meshing of the typical unit cell is performed as shown in Figure 3. All the finite elements of the cell have the common reference key point P_3 , which is located at the intersection of the axis of the cylinder and the plane of the circumferential rib. The use of this point as a reference allows the original natural twisted geometry of the curvilinear helical ribs to be closely approximated by the linear finite elements. Using rotation, copying, and translation of the meshed typical unit cell (see Figure 3), the complete finite element model of lattice cylindrical shell is generated. Then the ribs' characteristics are assigned including the sizes of the cross-sections and the material properties, i.e. moduli of elasticity and densities.

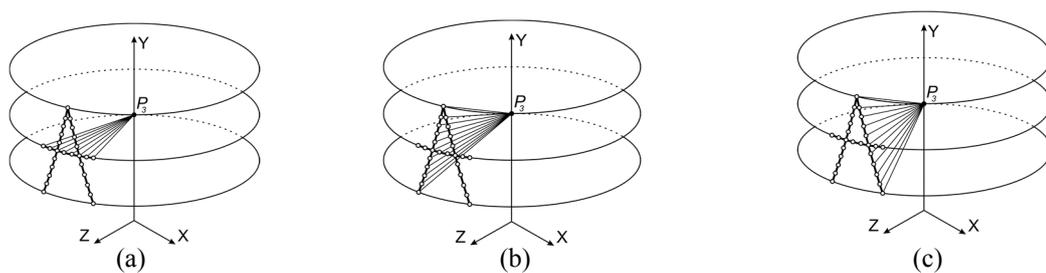


Figure 3. Meshing of the unit cell and positioning of the finite elements for the hoop ribs (a), and helical ribs with $+\phi$ (b) and $-\phi$ (c) angles of orientation.

The modelling algorithm has been realised in the form of the finite element model generator program and has been used for the automated finite element modelling of the lattice cylindrical shells.

Buckling analyses have been performed for the shells having a diameter $D = 1$ m and lengths $L = 0.5, 1.0$ and 1.5 m using the FEA package COSMOS/M. The shells are composed of helical and hoop ribs of a 5×5 mm square cross-section and made of unidirectional carbon-fibre reinforced plastic (CFRP) having the following properties: modulus of elasticity, $E_x = 100$ GPa, shear moduli, $G_{xy} = G_{xz} = 5.5$ GPa, $G_{yz} = 2.5$ GPa, and density, $\rho = 1500$ kg/m³. The analyses have been performed for a number of shells having the helical angles changing from 20° to 50° in the 5° increments. The number of helical ribs of one helical

direction (either $+\phi$, or $-\phi$) $n = 100$ has been kept the same for all the shells. The size of the beam finite elements was ranging from 10 to 20 mm. For instance, the lattice structure of the shell with the length $L = 0.5$ m and the angle $\phi = 20^\circ$ has been modelled with 10400 20-mm long finite elements. The finite-element model of the lattice shell having $L = 1.5$ m and $\phi = 50^\circ$ has been built of 81800 20-mm long finite elements. The bottom end of the shell is fully clamped and the axial load P is applied through the rigid ring attached to the top end. The critical loads $P_{cr}(L, \phi)$ obtained from the finite-element buckling analyses of the shells having various lengths and helical angles and subjected to axial compression are shown in Figure 4a.

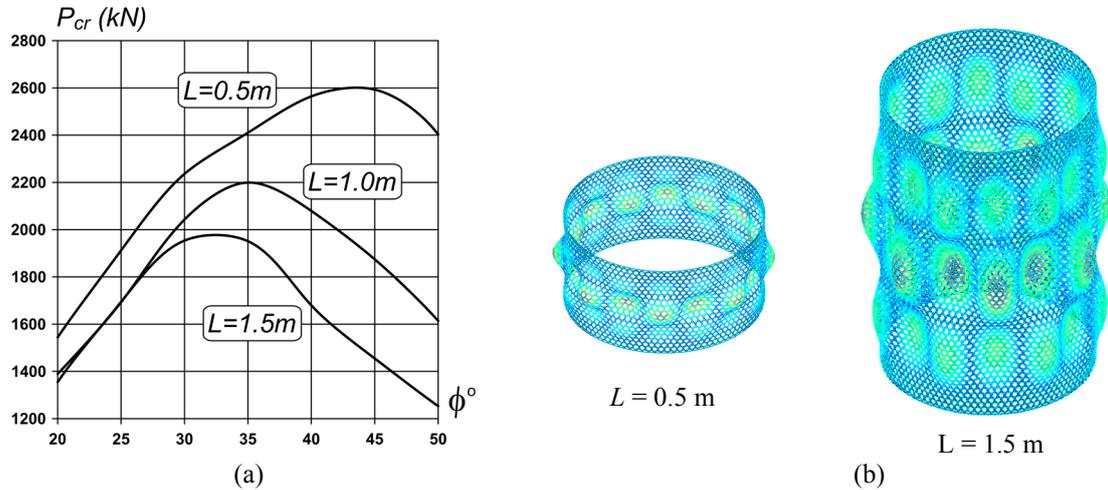


Figure 4. Function $P_{cr}(L, \phi)$ for the lattice shells under axial compression (a), buckling mode shapes for the shells with helical ribs wound at $\phi = 30^\circ$ (b).

Examples of the buckling mode shapes for the shells with helical ribs wound at $\phi = 30^\circ$ are shown in Figure 4b. Based on these computations, it was found that the shells with the helical ribs oriented at $\phi = 30^\circ$ deliver the maximum weight efficiency. More computational results of the buckling analyses of cylindrical lattice shells subjected to transverse bending, pure bending, and torsion can be found in Morozov et al. (2011a).

2.2. Conical lattice shells

Similarly to the cylindrical lattice shells, truncated lattice conical shells are composed of two symmetric systems of filament wound helical and circumferential (hoop) ribs as shown in Figure 5.

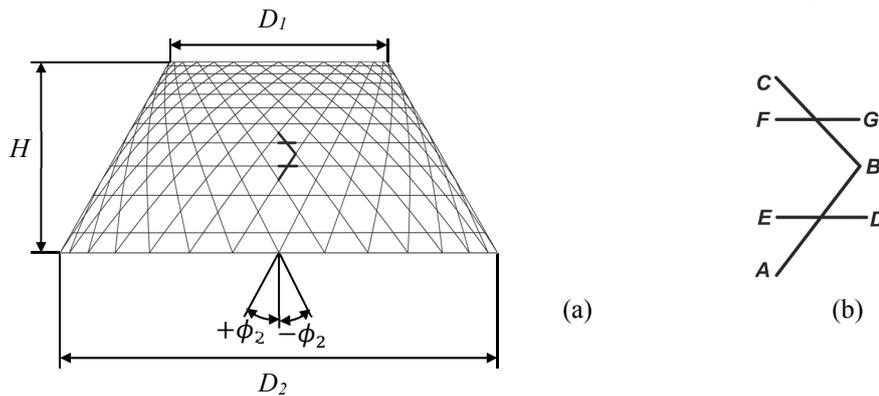


Figure 5. Conical lattice shell (a), and typical elementary unit (b).

The set of input data for the finite-element modelling includes: the height of the shell, H , diameters of the top and bottom sections, D_1 and D_2 , number of the helical ribs of one direction (either $+\phi$, or $-\phi$), n and the angle of the rib orientation at the bottom section of the shell ϕ_2 , (see Figure 5a). The finite-element model of the shell is generated using a typical elementary unit of the grid structure composed of two segments of the helical ribs AB and BC and two segments of the hoop ribs ED and FG as shown in Figure 5b. The lattice shell is modelled using the beam finite element referred to the local coordinate frame xyz as shown in Figure 2b. Changing the location of the point P_3 within the frame XYZ , the required orientation of the beam element can be achieved.

The typical elementary unit of the lattice structure (see Figure 5b) is meshed as shown in Figure 6a. All the beam elements of the unit $ABED$ have the common reference key point P_{3ED} whereas all the elements of the unit $BCFG$ have the common reference key point P_{3FG} . Both points P_{3ED} and P_{3FG} are located at the intersections of the cone axis Y with the planes of the hoop ribs ED and FG , respectively (see Figure 6a). The selected positions of the key points allow the real geometry of the lattice shell composed of the twisted helical ribs to be reliably presented in the model.

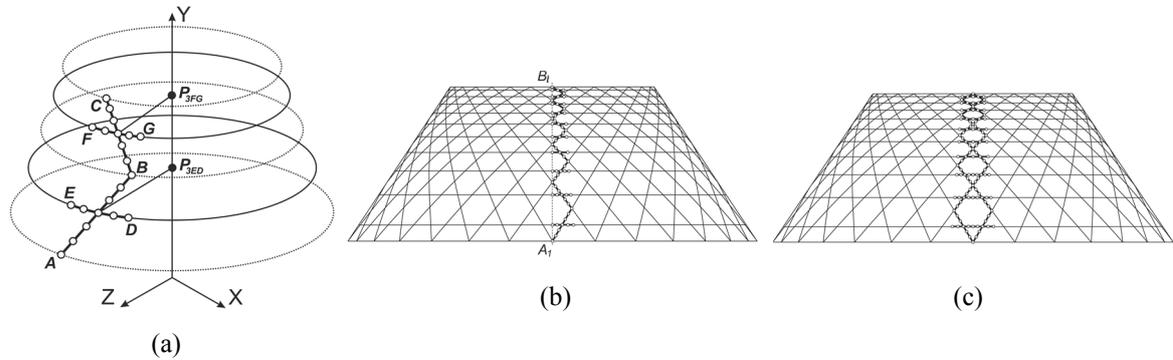


Figure 6. Meshing the elementary unit (a), placement of the units along the cone generator (b), basic finite-element cell of the lattice structure.

The actual construction of the finite-element mesh is performed starting from the unit located at the bottom of the shell (see Figure 6b). The meshing is then carried on for the units located on top of each other along the cone generator line A_1B_i . As shown in Figure 6c, i is the number of the last unit in this sequence located at the top of the cone. The helical ribs follow the geodesic trajectories which are defined by Clairaut's law: $r(Y) \cdot \sin \phi(Y) = const = (D_2/2) \sin \phi_2$, where $r(Y)$ is the current radius of the cone. The helical angle $\phi(Y)$ of the rib orientation changes according to this law from the bottom section to the top one. This change is taken into account when constructing the finite-element model from the elementary units as shown in Figure 6b. The meshed column group of these units shown in this figure is then mirror copied with respect to the cone generator line A_1B_i creating the basic finite-element cell of the lattice structure shown in Figure 6c. Using copying and rotation of this cell around the cone axis, the complete finite-element model of the lattice conical shell is automatically generated. Once this procedure is complete, the meshed top and bottom end rings are added to the model. Input of the geometry parameters and material properties of the ribs completes the construction of the finite-element model. This algorithm has been implemented in the form of a finite-element model generator program specifically developed for the problem under consideration. The generator was created using an internal FEA package code and has been applied to the automated construction of the finite-element models of conical lattice shells having various structural parameters (Morozov et al. (2011b)).

The buckling analyses have been performed for the filament wound composite conical lattice shells subjected to axial compression using COSMOS/M FEA package. The dimensions of the shells are: $H = 0.6$ m, $D_1 = 0.8$ m, $D_2 = 1.6$ m. The shells are made of the same unidirectional carbon-fibre reinforced plastic (CFRP) as the cylindrical shells considered earlier. The number of helical ribs of one helical direction (either $+\phi$, or $-\phi$) is $n = 60$. The analyses have been performed for the shells having the angles $\phi_2 = 5^\circ, 10^\circ, 15^\circ, 20^\circ$ and 25° . Examples of finite-element models created for the angles $\phi_2 = 5^\circ, 15^\circ$, and 25° are shown in Figure 7.

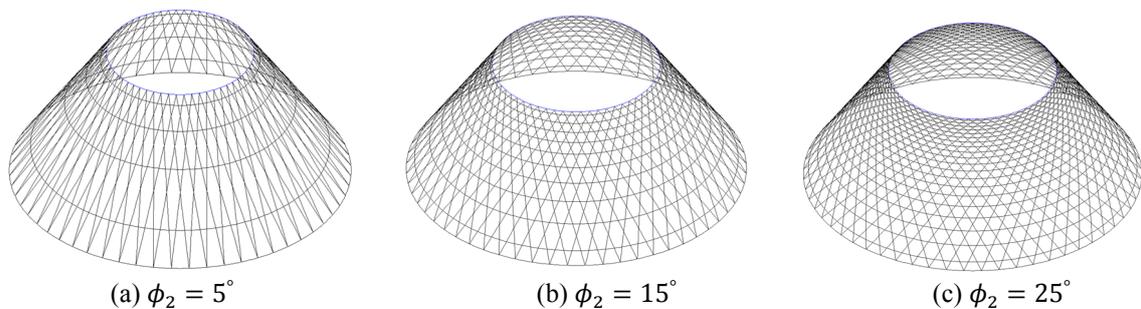


Figure 7. Finite-element models of the lattice conical shells.

It is assumed, that all the ribs have the cross-sectional area of 16 mm^2 , whereas the height-to-width ratio h/b (see Figure 2b) can vary. The following three combinations were considered in the analyses: (1) $h = 2 \text{ mm}$, $b = 8 \text{ mm}$, $h/b = 0.25$; (2) $h = 4 \text{ mm}$, $b = 4 \text{ mm}$, $h/b = 1$; (3) $h = 8 \text{ mm}$, $b = 2 \text{ mm}$, $h/b = 4$. The lattice shell structures are modelled using the beam finite elements with the length ranging from 3.5 to 20 mm depending on the angle ϕ_2 . For instance, the lattice structure of the shell having the angle $\phi_2 = 10^\circ$ has been modelled with 10800 10 to 20-mm long finite elements. The finite-element model of the lattice shell having $\phi_2 = 25^\circ$ has been built of 25200 3.5 to 15-mm long finite elements. The bottom end of the shell is fully clamped and the compressive force P is applied through the rigid ring attached to the top section. The critical loads $P_{cr}(h/b, \phi_2)$ obtained from the simulations of the shells having various ratios h/b and angles of the helical ribs' orientation at the bottom section, ϕ_2 are shown in Table 1.

Table 1. Buckling loads P_{cr} (kN) (axial compression).

ϕ_2°	$h = 2 \text{ mm}$ $b = 8 \text{ mm}$	$h = 4 \text{ mm}$ $b = 4 \text{ mm}$	$h = 8 \text{ mm}$ $b = 2 \text{ mm}$
5	13.863	34.416	44.334
10	32.969	70.541	110.002
15	60.601	109.475	179.714
20	96.801	150.397	252.123
25	139.542	192.213	316.574

As expected, the shells with the highest ratio h/b ($h = 8 \text{ mm}$ and $b = 2 \text{ mm}$) are the most resistant to buckling. Typical buckling mode shapes are shown in Figure 8. The shells with $h/b = 0.25$ and 1 buckle in the mode characterised by formation of a few buckle waves at the bottom section which is typical for conventional homogeneous conical shells (see Figure 8a). For certain combinations of the helical angles and dimensions h and b , shells buckle in the second mode characterised by the local buckling of ribs (see Figure 8b). This is typical for the lattice shells having $h/b = 4$ and the angles $\phi_2 = 5^\circ, 10^\circ, 15^\circ$ and 20° .



Figure 8. Buckling mode shapes of conical lattice shells: (a) $\phi_2 = 15^\circ$, $h = b = 4 \text{ mm}$; (b) $\phi_2 = 15^\circ$, $h = 8 \text{ mm}$, $b = 2 \text{ mm}$.

2.3. Parabolic composite lattice shells

The modelling approach presented in the previous sections was applied to the buckling analysis of a structure composed of parabolic composite lattice shells as shown in Figure 9.

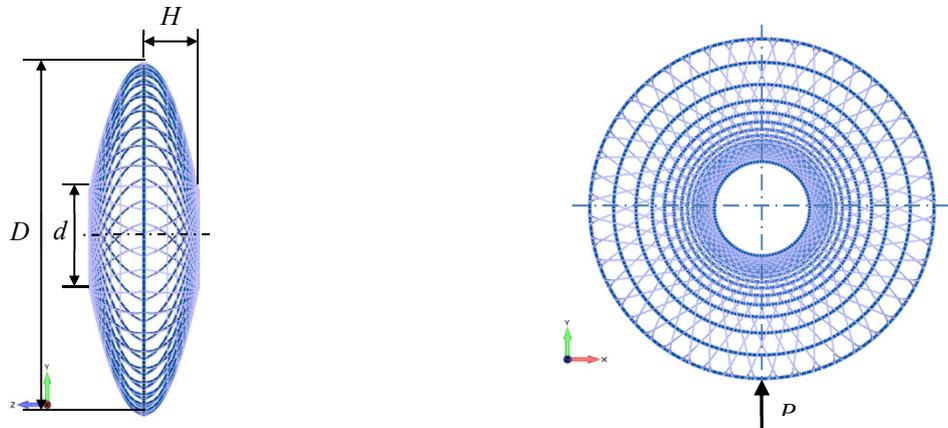


Figure 9. Parabolic composite lattice shell.

The shell is made of carbon fibre reinforced plastic by filament winding and loaded by concentrated load $P = 1000$ N as shown in Figure 9. The shell dimensions are: $D = 1$ m, $d = 0.139$ m, $H = 0.17$ m. The helical ribs have the following characteristics: $h = 8$ mm, $b = 2$ mm, $E = 215.5$ GPa, $G = 3.5$ GPa, $\rho = 1650$ kg/m³. The hoop rings have 10 x 2 mm rectangular cross sections and are made of the same material. The parabolic shells are joined by the internal and external rings with 10 x 10 mm square cross sections. The structure is fully clamped at the inner diameter. The value of critical buckling load obtained from the finite element analysis is $P_{cr} = 20.765$ kN. The corresponding buckling mode shape is shown in Figure 10.

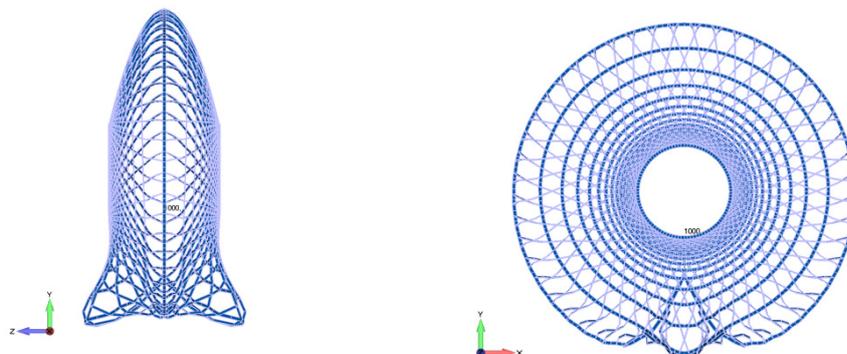


Figure 10. Buckling mode shape of the parabolic lattice shell.

3. CONCLUSIONS

A modelling approach based on the finite element discrete approximation is presented in this paper. The lattice shells are modelled as three-dimensional frame structures composed of curvilinear ribs subjected to the tension/compression, bending in two planes and torsion. The specialised finite-element model generation procedures developed in this work were applied to buckling analyses of composite cylindrical, conical, and parabolic shell structures. It is demonstrated that the simulations performed are capable of capturing major mechanical effects and predicting mechanical behaviour of composite lattice structures.

It was shown, based on the computational results that the modelling techniques considered in this paper provide effective tools for structural design and optimisation of composite structural components made in the form of filament wound lattice shells of various configurations. This is demonstrated for a number of particular structural applications.

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