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Parameter sensitivity analysis of the MERGE quasi-steady state gully erosion model

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Abstract: Gully erosion is the major source of sediment across the Great Barrier Reef Catchment Area (GBRCA) contributing an estimated 54% of the total sediment generated from landscape processes. It would stand to reason therefore that any effort to reduce the load of sediment delivered from the GBRCA to the receiving waters of the Great Barrier Reef would focus on the remediation of gully erosion to some extent. To this end, the Australian and Queensland Governments are directing significant investment on projects aimed at reducing gully erosion.

Mathematical gully erosion models can be useful for predicting the behaviour and evolution of gully initiation and growth to provide estimates of sediment export to the wider environment. Models can also be used to investigate various gully rehabilitation scenarios to quantify the potential mitigatory impacts of proposed projects and to help optimise their outcomes. MERGE (Modelling Erosion Resistance for Gully Erosion) is one such model that can quantify sediment loads resulting from gully erosion processes. MERGE is process-based and can be used to model the impacts of actions taken to rehabilitate gullies and reduce erosion to inform decisions related to gully remediation projects to protect coastal ecosystems.

To gain some insight into the behaviour MERGE we conduct global sensitivity analyses on a range of model gully scenarios with distinctive characteristics. Sensitivity assessment has become a standard procedure for exploring the factors influencing model output quantities of interest. The credibility and utility of MERGE depends on knowledge of how important each variable (parameter and model input) is. The objective of sensitivity analysis is to quantify the incremental change in model response to incremental change in model variables.

We find that the relative sensitivity of MERGE model parameters can vary significantly between different sce-narios. A practical consideration arises from observing how the hierarchy of parameter influence on model response changes depending on the scenario. For example the more important variables may need refine-ment, and identifying those with little influence on model outputs may lead to model structure improvements and possibly simplifications. Furthermore, improved understanding of which gully process dominate the gen-eration and transport of sediment may assist in designing engineering solutions for limiting erosion in the environment.

Keywords: *Erosion, gullies, intervention, process-based model, sediment*

1 INTRODUCTION

Gully erosion is understood to be a significant source of sediment being discharged from the landscape into the receiving waters of the Great Barrier Reef lagoon. Although most gully erosion is localised within parts of the Burdekin and Fitzroy catchments it is responsible for an estimated 40% of sediment entering the waterways of the GBR catchments (McCloskey et al. 2021). The MERGE gully erosion model was developed to support gully remediation activities to achieve sediment reduction targets for the Great Barrier Reef. The sensitivity analysis of MERGE model parameters is an important preparatory step to support use of the model to guide decision making by land managers.

It has been noted elsewhere that sensitivity analysis should be conducted on gully models to inform their use (Roberts et al. 2022). In this paper we use the method of Sobol (Sobol 2001) to survey the behaviour of the MERGE model in terms of the sensitivity of the calculated rate of sediment discharge to different gully conditions as represented by different model parameter configurations.

Sobol sensitivity analysis is a powerful and widely used technique for understanding the sensitivity of model outputs to variations in input parameters. It is a variance-based method that is designed to measure the contributions of individual input parameters and their interactions to the overall variance of the output. The method was first introduced by Sobol in 1993 (Sobol 1993) and has since become a standard tool for sensitivity analysis in many fields, including engineering, finance, and environmental science.

The basic idea behind Sobol sensitivity analysis is to decompose the total variance of the model output into contributions from each input parameter and their interactions. This decomposition provides a quantitative measure of the sensitivity of the model output to each input parameter, and it can be used to rank the importance of the inputs and identify the most influential ones. The method is particularly useful when dealing with high-dimensional models that have a large number of input parameters, as it allows for the efficient exploration of the parameter space.

Sobol sensitivity analysis has several advantages over other sensitivity analysis methods. For one, it can handle non-linear and non-monotonic relationships between inputs and outputs, as well as interactions between inputs. Additionally, it can provide a measure of the total variance of the model output, which can be useful for uncertainty quantification and risk assessment.

2 METHODS

2.1 MERGE

MERGE is a one-dimensional conservation of mass model for the erosion of channel-like gullies (Figure 1). Sediment is advected through the gully subject to deposition (sink) and entrainment (source) processes, which are themselves functions of the sediment concentration C [kg/m^3]. MERGE incorporates a depositional layer and the concept of re-entrainment, where recently deposited sediment is easier to entrain than the original soil matrix. A depositional layer will form if the rate of entrainment is less than the rate of deposition, which is typically observed when there is a high concentration of sediment within the water column due to up-gully conditions, but a low power environment. For convenience, the gully is divided into two sections, the gully head, which is a higher power environment due to the waterfall and within which a depositional layer cannot form; and the gully channel, which is a lower power environment.

MERGE assumes a homogeneous gully, with constant soil properties (soil cohesion J [Ws/kg]), density σ [kg/m^3], and particle radius R [m]) and constant channel properties (head length L_w [m], total length L [m],

Figure 1. Examples of linear gullies within the GBR catchments. Images © 1– R Thwaites; 2 & 3 –A Brooks

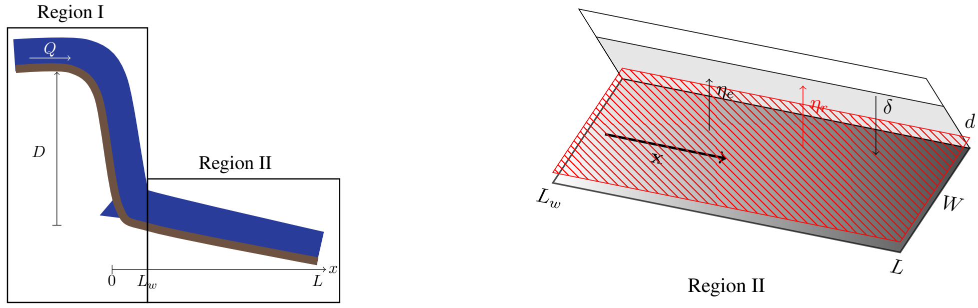

Figure 2. Geometry of the ideal gully. The sediment concentration $C(x,t)$ [kg/m^3] satisfies the conservation of mass equation, which is to be solved within the bounded domain $x \in [0, L]$, split into two regions for convenience. Region I encompasses the gully waterfall $[0, L_w)$, while Region II the gully channel $[L_w, L]$. The gully is of constant slope S. The flow within the gully is of constant flux Q with depth d [m], and width W [m]. Sediment is entrained from the walls and floor at rate η_e [kg/m s] and re-entrained at rate η_r [kg/m s]. Sediment is deposited out at rate δ [kg/m s] potentially forming a depositional layer (shown in red). Figure adapted from Roberts (2019).

width W [m], slope S, and channel roughness n [s/m^1/3]) (Figure 2). Under the assumption of a constant flow, Q, through the gully, and hence a constant flow depth d, quasi-steady state solutions exist. The quasi-steady state solutions provide a steady concentration profile within the gully, however the dynamics of any depositional layer, if present, will not be steady. For a complete description of the model, refer to Roberts (2020).

The sensitivity analysis is conducted over five different gully and flow configurations (see Table 1). The scenarios are motivated by previous gully studies with modifications to sample uncertainty across a broader range of gully erosion conditions. Scenarios 1 and 2 are motivated by the Bremer gully reported in Rose et al. (2014), with Scenario 1 modified for smaller particle sizes and density, and Scenario 2 considering a lower soil cohesion and channel roughness. Scenario 3 is considers the gully reported in Prentice et al. (2021), while Scenario 4 considers the first section of the gully reported in Roberts (2022). Scenario 5 examines the conditions from the first section of the Riverside North Gully reported in Prentice et al. (n.d.).

2.2 Random Sampling High Dimensional Model Representation

Consider $\mathbf{x}= (x_1, x_2, \cdots, x_n)$, which can represent the parameters of a function, or model, with some response $f(\mathbf{x})$. The parameters $\mathbf{x}$ take values in the unit hypercube $[0,\ 1]^n$ after being rescaled from their original values. The ANOVA-HDMR (analysis of variance - high dimensional model representation) of $f(\mathbf{x})$ expands the function into the sum of component functions in the following form

$$f(\mathbf{x}) = f_0 = \sum_{i=1}^{n} f_i(x_i) + \sum_{1\leq i\leq j\leq n} f_{ij}(x_i, x_j) + \cdots + f_{12\cdots n}(x_1, x_2, \cdots, x_2). \quad (1)$$

The partial variances attributed to each variable and combination of variables can be evaluated by calculating the variance of their respective component functions over [0,1]. In order to symbolically derive the ANOVA-HDMR and variances, the function $f(\mathbf{x})$ must be square integrable. If this is not the case, for example when $f(\mathbf{x})$ is the response of a black box model, numerical Monte Carlo integration can substitute for analytic integration. This *brute-force* approach can require a large number of function evaluations, which can be problematic in the presence of an expensive model. The random sampling HDMR (RS-HDMR) is a more efficient alternative to the ANOVA-HDMR that proceeds by expanding the function variables into an orthonormal basis and approximating the HDMR component functions as a sum of orthonormal polynomial basis functions as follows:

$$f_i\left(x_i\right) \approx \sum_{r=1}^{k} \alpha_r^i \varphi_r\left(x_i\right) \tag{2}$$

$$f_{ij}\left(x_i, x_j\right) \approx \sum_{p=1}^{l} \sum_{q=1}^{l'} \beta_{pq}^{ij} \varphi_p\left(x_i\right) \varphi_q\left(x_j\right) \tag{3}$$

where *k*, *l* and *l'* are the maximum orthonormal polynomial orders while α_r^i and β_{pq}^{ij} are constant coefficients that can be determined by regression analysis. Eqs (2) and (3) show the RS-HDMR terms up to a second order expansion, which is usually sufficient in most cases to capture the most dominant terms in the sensitivity analysis. The decomposition coefficients can then be used to estimate the partial variances for the first order terms

$$D_i = \sum_{r=1}^{k} \left(\alpha_r^i\right)^2 \tag{4}$$

and second order terms

$$D_{ij} = \sum_{p=1}^{l} \sum_{q=1}^{l'} \left(\beta_{pq}^{ij}\right)^2 \tag{5}$$

and finally, the first order Sobol indices can be calculated

$$S_i = \frac{D_i}{D} \tag{6}$$

and second order indices

$$S_{ij} = \frac{D_{ij}}{D} \tag{7}$$

where *D* is the total variance of the response represented by $f\left(\mathbf{x}\right)$.

2.3 RS-HDMR implementation

To investigate the evolution of parameter sensitivities as a function of varying gully characteristics, 5 different parameter configurations were constructed as outlined in Table 1. 11 model parameters were considered for each configuration, namely gully width W, head length L_h, head depth D_0, slope S, Manning's roughness coefficient n, flow depth d, particle radius R, soil density σ, soil cohesion J, carrying capacity C^*, and power proportion k, which defines the proportion of power (stream and waterfall) available to erode the floor and walls. The volumetric flux through the gully Q is related to the flow depth, gully slope, and width using Manning's Equation. The settling velocity of the sediment is modelled using Stoke's Equation and is a function of the particle size and density. The sediment yield is measured at the distance $L = 100$ m from the start of the gully. All other parameters are held constant. Latin hypercube sampling (LHS) (Deutsch & Deutsch 2012, Moza 2020) was used to generate 1000 uniformly distributed parameter sets centred at the values in Table 1 with ranges extending $\pm 30\%$ from the mean for each scenario. The MERGE model was run for each parameter set to generate the steady state sediment discharge rate response variable. Infeasible parameter combinations were discarded. Once the Latin hypercube is scaled to the unit hypercube, the scaled parameters can be expanded into a shifted Legendre polynomial basis. The n^{th} shifted Legendre polynomial can be found using a variation of Rodrigues formula (Horner 1965):

$$\varphi_n\left(x\right) = \frac{\sqrt{2n+1}}{n!} \frac{d^n}{\mathrm{dx}^n} \left(x^2 - x\right)^n \tag{8}$$

and they obey the orthonormality condition,

$$\int_0^1 \varphi_n\left(x\right) \varphi_m\left(x\right) = \delta_{mn}. \tag{9}$$

Table 1. MERGE model parameter values and sensitivity indices (*Si*) for the five gully scenarios. Model sensitivities are based on uniform LHS sampling centred at Value as described in the text. Only indices $>$0.01 are explicitly provided. Second order indices follow the first order indices in the table. For reference, the mean and standard deviation of the model response for each scenario is given in the final row.

	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Scenario 5	
Parameter	Value	S_i	Value	S_i	Value	S_i	Value	S_i	Value	S_i
W	2	<0.01	2	0.01	5.5	<0.01	2.4	0.09	1.25	<0.01
L_h	20	<0.01	20	<0.01	1	<0.01	3.1	0.10	1.5	<0.01
D_0	3	0.21	2.2	<0.01	2	0.25	2	<0.01	0.4	<0.01
S	0.01	0.02	0.02	0.11	0.012	<0.01	0.025	<0.01	0.055	0.29
d	1.5	<0.01	0.5	0.01	0.584	<0.01	1.3	0.39	0.34	<0.01
n	0.2	<0.01	0.04	0.05	0.027	<0.01	0.045	0.01	0.027	<0.01
k	0.2	0.37	0.2	0.03	0.2	0.32	0.005	<0.01	0.01	0.32
C^*	472	<0.01	472	0.23	266	<0.01	147	<0.01	133	<0.01
R	16μm	<0.01	65μm	0.20	16μm	<0.01	10μm	0.40	16μm	<0.01
σ	1330	<0.01	2320	0.32	1330	<0.01	1470	0.01	1330	<0.01
J	400	0.40	5	<0.01	100	0.35	1700	<0.01	146	0.35
J, k		0.01		<0.01		<0.01		<0.01		0.01
C^*, σ		<0.01		0.01		<0.01		<0.01		<0.01
Mean yield (SD) [kg/s]	19.7 (5.4)		301.4 (80.4)		69.1 (16.3)		0.07 (0.02)		5.8 (1.7)	

The coefficients, α_r^i and β_{pq}^{ij} can now be found by regression analysis of Eq. 10.

$$f\left(x_1, \cdots, x_M\right) \approx c_0 + \sum_{i=1}^{M}\sum_{r=1}^{k}\alpha_r^i\varphi_r\left(x_i\right) + \sum_{1\le i\le j\le M}^{M}\sum_{p=1}^{l}\sum_{q=1}^{l'}\beta_{pq}^{ij}\varphi_p\left(x_i\right)\varphi_q\left(x_j\right) \tag{10}$$

For this analysis, $k = l = l' = 10$ was used for the basis set expansion. When $M = 11$ a total possible 6105 coefficients are included in Eq. 10. To limit the number of coefficients in the fitting procedure a Group Method of Data Handling (GMDH) polynomial neural network regression model (Ivakhnenko 1971) is initially constructed using the primitive Legendre polynomial basis set in a similar way to that outlined in Lambert et al. (2016) and Bennett & Fentie (2017) . Terms selected in the best performing neuron in the GMDH network are then selected and the remaining primitive basis set variables are discarded. The filtered basis set that survives the GMDH filtering step are then combined to form all possible first and second order product terms to form a pruned version of Eq. 10. The related subset of coefficients can then be found through a more manageable analysis using automatic relevance determination (ARD) regression (Rudy & Sapsis 2021). This composite GMDH/ARD procedure can yield a sparse RS-HDMR, which can be thought of as a type of polynomial chaos expansion (Blatman & Sudret 2010). The method described in this section is implemented in a Python package, RSHDMRPy, which can be made available on request.

3 RESULTS AND DISCUSSION

Table 1 provides the results of the sensitivity analysis for each of the 11 parameters across the five scenarios investigated. Across all scenarios 4 of the 11 parameters accounted for at least 30% of the observed variation in at least one of the scenarios, with an additional 3 parameters accounting for at least 20% of the observed variation. Second order indices account for at most 1% of the total model variance so can generally be regarded as non-influential for our purposes. The relative sensitivity of the parameters was not consistent between the different scenarios, suggesting that uncertainty in specific parameters will have greater importance in some cases than others.

Scenarios 1 and 3 had the same dominant parameters: the soil cohesion J, power proportion k, and waterfall height D_0. In these scenarios the model is most sensitive to the parameters driving the rate of entrainment in the gully head, which is the region of highest entrainment in most systems. Entrainment in the gully head drives headcut retreat, the rapid lateral expansion of a gully up the catchment. Entrainment is a balance between the soil's resistance to erosion, where soil cohesion is a critical parameter, and the power that is available to entrain. Within the gully head power from the waterfall is typically much greater than that from the streamflow. The waterfall power is driven by the height through which the water falls over the head, $D_0 - d$, and the volumetric flux of the water, Q (captured in this analysis by d since flux is related to depth by Manning's Equation). The waterfall height D_0 is measured in the field, however some natural variation throughout the head region is expected. The power proportion $k < 1$ is a multiplicative factor that restricts the amount of power from the flow that is available for erosion – the value of k is unknown. Roberts (2020) assumed that k was constant across systems, however more recent studies suggest that this may not be the case.

Scenario 5 yielded similar sensitivities to Scenarios 1 and 3, however the slope S accounted for 29% of the observed variability, whereas the waterfall depth accounted for $< 1\%$. In this scenario, the waterfall depth was small $D_0 = 0.4$ with a flow depth of $d = 0.34$, while the channel is steep $S = 0.055$. In this scenario stream power exceeds waterfall power leading to the model being more sensitive to the driver of stream power (slope) balanced against the power proportion k and soil cohesion J.

Scenarios 2 and 4 exhibited different patterns of sensitivity to 1, 3 and 5. Variation in Scenario 4 is dominated by the particle size (40%) and the flow depth (39%). Scenario 2 is the highest yielding with a mean sediment yield of 301.4 kg/s, with the next closest being Scenario 3 at 69.1 kg/s. The variation in the sediment yield is mostly explained by the soil properties of density (32%) and particle size (20%), and the carrying capacity (23%). This scenario considered a large, heavy soil ($R = 65\mu m$, $\sigma = 2320$) with low cohesion ($J = 5$). Therefore, deposition is likely an important process in this scenario, and thus variation in the settling velocity, which is a function of the density and particle size, is likely driving this uncertainty/variation.

In most scenarios the carrying capacity has a limited effect on the variation ($< 1\%$), with the exception of Scenario 2. The carrying capacity limits the rate of entrainment due to the concentration of sediment already in the water column, C, by a factor of $1 - C/C^*$. Thus, if $C \ll C^*$ it has a negligible effect on the entrainment process, but as $C \to C^*$ it reduces the rate of entrainment to zero. It is therefore not unexpected that the carrying capacity will have a strong effect on variation for scenarios where the concentration approaches the carrying capacity, and a negligible effect otherwise. The carrying capacity is thought to be a global value, possibly related to the sediment density, but is unknown.

With the exception of the steep slope in Scenario 5, these scenarios are insensitive to the measured gully characteristics of slope, S, width, W, and head length L_h. This indicates that MERGE is not likely to be very sensitive to how a gully system is delineated into one-dimensional sectors (see Roberts (2022) for an example of segmenting a gully for modelling). If the slope is moderate and variation in the slope and width $< 30\%$, the section can likely be captured by a single set of parameter values.

One limitation of this study is the use of the ±30% bound in variation across all parameters. This choice is reasonable for characteristics such as width, depth, flow and roughness, given the typical approach to dissect a gully into approximately homogeneous sections (see for example Roberts (2022), Prentice et al. (n.d.)). However, it is less realistic for sediment characteristics where a fixed rather than percentage error would better reflect measurement error and natural variation in a system.

This sensitivity analysis reinforces previous calls for further research to understand the currently unknown soil cohesion, power proportion, and carrying capacity parameters and how these should be selected in MERGE (Prentice et al. 2021, Roberts 2022).

4 CONCLUSIONS

There are some practical outcomes that may be drawn from this study. The results show that the model parameter importance can vary significantly depending on the scenario. From a modelling perspective, insensitive parameters that contribute negligibly to variance in the model response can often be disregarded for calibration purposes since they do not provide useful information. Care must be taken, however, not to assume that a parameter that can be neglected in one scenario is not an important contributor to model output variance in another, and *vice-versa*.

Gully model parameter sensitivities may also provide a useful reference point for informing decisions about engineering solutions for gully rehabilitation.

Finally, the RS-HDMR is a robust regression model that is fully explainable and can be used as a surrogate for the primitive MERGE model.

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