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A review of UAV swarm-based detection of a dynamic contamination plume

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Abstract: Developing a capability to track a contamination plume is desirable within both defence and civilian contexts. Example scenarios could include defence humanitarian and disaster relief missions where assistance is required in an area where contaminants have been released into the air as part of a natural disaster, e.g. the Fukushima nuclear disaster in 2011. Contaminants could be chemical, biological, radiological or nuclear. Here, tracking a dynamically moving contamination cloud or plume is essential to understand safe areas of operation.

One proposed solution to this challenge is the use of a swarm of uncrewed aerial vehicles (UAVs or drones), carrying appropriate sensors, to locate and dynamically track the contamination plume. Developing such capability is challenging, and one critical step to achieve this is the development and use of an appropriate modelling and simulation capability. Drivers for this simulation based development approach include; the practical challenges for producing a contamination plume for field evaluation of solutions, the complexity involved in developing control mechanisms for a UAV swarm, and the environmental variability which make rapid field evaluation challenging and inconsistent. In this paper, we review current literature associated with modelling and simulation of this capability and propose a way ahead from the identified options.

To develop, simulate and test a plume tracking capability as proposed, multiple interrelated components need to be developed. Firstly, an appropriate model of contamination plumes must be developed. This can be used to stimulate the UAV mounted sensors which feed measurements to plume tracking algorithms to collaboratively develop an estimation of the contamination plume. This estimated contamination plume can then bias the control of the UAVs towards regions of higher concentration, hence containing the UAVs in the plume and continuing to sense the contamination. This forms a control loop, adapting the position of the UAVs to track the movement of the plume. We review the current literature for each of these components of a plume tracking system, identify shortcomings or gaps in the existing approaches, and propose an end-to-end solution for modelling, stimulating and simulating this capability. The key findings were as follows;

It was identified that there is limited research that presents a fit for purpose model of a contamination plume to stimulate the sensors and control components of a UAV swarm solution. Hence, we investigate this challenge and identify the range of plume modelling approaches. For our simulation purposes, we assess that a Lagrangian approach provides an appropriate balance of plume fidelity with low simulation overheads.

When reviewing plume mapping algorithms for a distributed mobile sensor network, we found that limited existing research addresses the dynamic nature of a moving contamination plume. However, that is required for our application.

Similarly, the coverage control component, which guides and controls the location of the UAVs, typically address static scenarios when producing optimal control solutions. Some dynamic coverage control approaches are identified but further work is required to provide stability guarantees when coupled with specific plume tracking algorithms.

Once combined and integrated, these components will form a suitable simulation environment and implementation approach for UAV based contamination plume tracking solutions.

Keywords: *UAV, plume modelling, swarm, coverage control, field estimation*

1. INTRODUCTION AND BACKGROUND

The capacity to remotely detect and map a contamination plume, potentially with chemical, biological, radiological, and nuclear contaminants, is an area of interest to both civilian and defence groups. This could enable the remote identification of situations where the use of personal protective equipment is necessary or alternative routes are needed to avoid the contaminant. Similarly, this technology would be useful for first emergency responders and other civilian agencies to assess industrial emissions and evaluate air quality (Rosser et al. 2015). Since the early 1990s, remote sensing using mobile robots has emerged as a prominent research area. Several works have leveraged networks or "swarms" of robots to improve the efficiency and efficacy of mapping algorithms (Egorova et al. 2016; Silic & Mohseni 2019; He et al. 2019), with the latter two validating their approaches through experimentation. Work conducted in this field has led to several research directions, most notably in areas such as "gas distribution mapping" and "source localization" where significant progress has been achieved (Gongora et al. 2020). Early studies of gas source localisation (GSL) focused mostly on developing gradient-based contamination algorithms for tracking contamination plumes back to their source. The development of these algorithms was significantly impacted by source finding techniques that can be seen in nature. While these algorithms were successful in identifying contamination sources in highly controlled conditions, they often failed in dynamic environments. An important factor to consider is that gas plumes are propagated by turbulent wind. Even minor turbulence can disperse the gas into discrete packets. This makes the concentration gradient far from a smooth gradient, but intermittent, and with substantial concentration variations. Therefore, it is difficult to obtain accurate concentration measurements with available sensing technologies. Sensors with slow response times will cause even broader problems. Alternatively, probabilistic GSL models have demonstrated improved performance in more realistic situations. However, a limitation of current probabilistic approaches lies in the atmospheric transport and dispersion models that they use, which often do not accurately represent real plume structures, especially in turbulent conditions. Several review papers have been published on the topic of GSL using mobile robots; Lilienthal et al. (2006) discuss the initial investigations on the subject. Chen & Huang (2019) and Ishida et al. (2012) offer overviews of commonly used biologically inspired algorithms, as well as an introduction to probabilistic approaches, and Francis et al. (2022) cover the latest development in probabilistic GSL algorithms.

Compared to the extensive work on GSL, gas distribution mapping (GDM) has received limited consideration. In addition to the temporal fluctuations of turbulent plumes, it is crucial to consider that gas sensors only provide measurements about a limited area of the plume. As a result, measuring the instantaneous concentration field without using a highly concentrated network of sensing devices is challenging. However, understanding the time-constant structure of a plume distribution can be more essential than pinpointing the precise source location. For instance, it can help identify areas with high concentrations of hazardous contamination and enable better risk management. Pervious work on GDM can be categorized into two groups: model-based approaches and model-free (statistical) approaches (Francis et al. 2022). A major restriction of the model-based approaches is that they primarily consider steady air flow without taking into account its direction and velocity. In realistic environments, however, time-varying airflow is much more common. Moreover, model-free distribution mapping algorithms can be used without assuming steady and uniform airflow, these algorithms will play an important role in decreasing the dependence of GDM algorithms on time-average dispersion models, Section 3 of this paper, will cover related work in this area, it will highlight the challenges that have been encountered and explore how we can address them.

For our application in developing a UAV based contamination tracking capability, we believe it is necessary to employ multiple interrelated components to overcome these challenges. We have not identified any comprehensive work that covers all relevant aspects. We suggest the initial step in achieving this goal is to develop an appropriate plume simulation model. This is not addressed in the existing literature reviews. Creating a simulation model that accurately reflects real-world conditions allow us to evaluate and compare various gas plume mapping algorithms, under the same conditions. This will also enable us to test and refine the plume mapping system in a controlled environment, before conducting a field test. However, the existing work does not fully address the challenges of the plume modelling in the simulation. Therefore, in Section 2, we aim to fill this gap by defining the plume structure, examining current modelling approaches, and selecting the most promising one for future investigation. Once an appropriate plume model has been developed, the next step is to select the appropriate mapping algorithm for evaluation in the simulation. Section 3 provides a summary of the existing plume mapping algorithms by analysing the available literature reviews on this topic. We evaluate the strengths and weaknesses of each of the algorithms. Based on this analysis we select the most appropriate algorithm for future work. After developing the simulation capability and selecting the appropriate mapping algorithm, the next step is to design a control mechanism for the drone swarm. This will enable efficient and effective coordination of the UAV's during plume tracking, including the ability to replace UAVs.

Section 4 discusses the complexities involved in developing control mechanisms and provide practical consideration of plume mapping. Section 5 presents the conclusion of the paper and proposed future work.

2. THE STRUCTURE OF THE PLUME

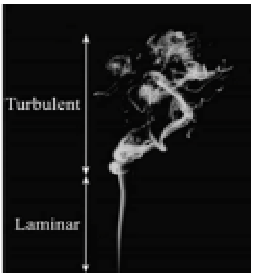

Figure 1. Transition from laminar flow to turbulent (Salmon 2015)

It is possible to gain an understanding of the structure of smoke plumes if we observe them from tobacco pipes. The end of the pipe heats up the air near the tip and then carries particles up that we call smoke; after leaving the source, they rapidly widen and then expand at a smooth and steady rate to form a discrete cloud without a fixed shape. Fig. 1 clearly demonstrates that initially laminar rise of hot air plume from a pipe transition to disordered, turbulent, motion of air a few centimetres away from the tip of the pipe. As the smoke rises, it accelerates because the warm air around it is less dense than the surrounding air, which create natural convection as it continues to rise and its velocity increases, it will eventually reach a threshold where its flow becomes turbulent. Up close, the smoke can be seen to contain thousands of filaments swirling in the wind with eddies at the edges. In the following section, we will delve into the work on plume simulation. One of the main reasons for developing simulated plume model is to evaluate and compare various contamination plume tracing algorithms, under the same conditions. This is because environmental parameters, such as wind, humidity, and temperature, greatly affect their performance. Simulation frameworks can also be used to test algorithms under different environmental conditions (strong airflows, turbulence, plumes, etc). Creating a simulation model that accurately reflects real-world conditions is a significant challenge, particularly when dealing with the intricate nature of contamination plume dispersal.

2.1. Atmospheric dispersion model

The transport of plumes in the atmosphere is primarily driven by wind (advection), and influenced by several other processes such as turbulent diffusion, deposition, chemical reactions, radioactive decay (where applicable)and physical transformations. These processes can be defined mathematically through the atmospheric transport equation.

The complexity of atmospheric dispersion models depends on various factors such as the spatial scale of the dispersion, the weather, and the terrain. The choice of a model depends on the purpose and scale of the simulation, as well as computational resources. Models range from local scale models with simple assumptions to global models with detailed physics. Dispersion models need meteorological data to simulate plume transport processes. There are two ways of coupling these models with numerical weather prediction (NWP) models: offline (using precomputed meteorological fields) and online (simulating both meteorology and dispersion simultaneously and optimizing NWP to the release site). Atmospheric transport equation can be solved in an analytic, numerical, and stochastic ways, which yields the Gaussian, Eulerian and Lagrangian models (Leelossy et al. 2018).

<u>Gaussian models:</u> Under the assumption of a single point source and uniform steady wind conditions, the analytical solution of atmospheric transport equation yields a normal distribution for the concentration field. Sutton 1953 proposed a Gaussian distribution model to describe gas plumes. The main advantages of Gaussian models are their short computation times and small input data requirements. A plume can be calculated by entering only a few parameters, making Gaussian models an effective tool for making decisions during emergency situations. However, the time-averaged Gaussian model do not capture short-term signatures of concentrations, the instantaneous peak concentrations or intermittent characteristics of plumes at short timescales (Farrel et.al. 2002).

<u>Eulerian models:</u> Eulerian models consist of a system of second-order partial differential equations (PDEs), in which space and time are the independent variables.The quantity of the PDEs in this system is determined by the number of chemical species present in the plume. The solution of the system provides plume concentration as a function of both space and time. Due to spatial and temporal variations in wind velocity, these PDEs cannot be solved analytically. This led to the development of several powerful numerical methods. One of the most common techniques is the method of lines. Solving Eulerian models in 3D can be computationally intensive. Adaptive gridding and parallelization are two of the most efficient ways to reduce the execution time. CMAQ, EAMC, EURAD are the examples of existing tools that use the Eulerian models (Leelossy et al. 2018).

<u>Computational fluid dynamics (CFD) simulations</u>: When atmospheric flow interacts with surface obstacles, its velocity profile undergoes significant changes, which can cause the spatial distribution of the plume to differ significantly from that over a flat surface. The NWPs models do not have the required resolution to accurately

represent these changes, so it is necessary to use CFD simulations. In CFD simulations, Naiver-Stokes equations are solved on a fine grid to accurately capture the microscale wind and turbulence field. This allows for a more accurate simulation of the plume concentration distribution between surface obstacles. However, determining the appropriate mesh size, boundary conditions, and turbulence model can be a complex and time-consuming task. There are various CFD software packages available for atmospheric simulations, including ANSYS Fluent and the open-source OpenFOAM model.

Lagrangian models: Lagrangian particle dispersion models (LPDMs) track the motion of individual particles to model the evolution of a plume as it disperses in the atmosphere. The particle's location is the dependent variable and the particle motion is solved using a stochastic equation. LPDMs describe plume particle movement using stochastic ordinary differential equations (ODEs) with both advection and turbulent diffusion affecting particle velocity. Advection velocity is from NWP model data but small-scale wind fluctuations are captured using Langevin's equation (a Markov process). Lagrangian models simulate particle motion and output their 3D coordinates over time. Concentration is calculated by summing particles in a selected volume around a receptor in a grid. Complex chemical mechanisms can be challenging to solve with this method since concentration is not a dependent variable. The computational cost of Lagrangian models increases with the number of particles but even a few trajectory calculations can provide important information on dispersion direction. Lagrangian models have hybrid with both the Eulerian and Gaussian models. An example of a hybrid Lagrangian-Eulerian approach is DREAM, which starts with a Lagrangian model at the local scale and gradually transitions to an Eulerian model at the large scale. Fig. 2 provides a visual representation of a plume structure at different temporal scales.

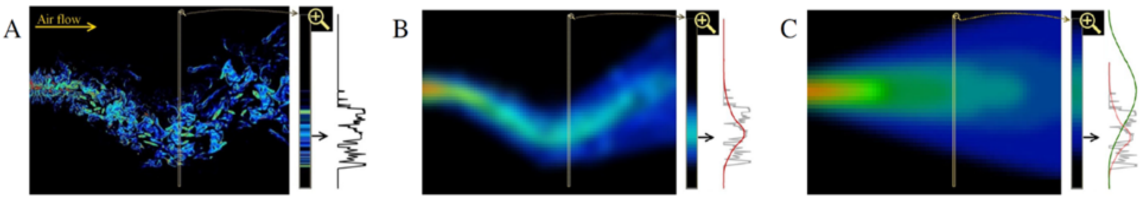

Figure 2. Visual representation of plume structure at different temporal scales. (A) shows the instantaneous structure (Eulerian model and Lagrangian model); (B) shows the average spatial distribution of plumes; (C) represents the average time and spatial distribution of plumes (Gaussian model) (Marjovi & Marques 2014)

The black trace in Fig. 2 A depicts the real-time readings of a rapid gas sensor during cross-wind movement, the red trace in B represents the readings of a slower sensor (which functions like a low-pass filter) moving cross-wind, and the green trace in C shows the average of the readings over an extended time period.

Leelossy et al. 2018 conducted a study on atmospheric dispersion modelling software and provided an overview of the available options. Their work highlights the different features and capabilities of each software, making it a useful reference for those looking to select the best tool for a specific application. Each of the mentioned modelling approaches has their own advantages and disadvantages. It is more feasible to use a Lagrangian model for evaluating plume tracking algorithms, as high-fidelity Eulerian models are computationally expensive. In addition to the above models, there exist approximated models such as the filament based model (Farrel et.al. 2002) that offer a cost-effective solution for simulating dynamic 2D concentration fields (see open-source PomPy model from GitHub - InsectRobotics/pompy). This model captures the key features of real plumes, including short-term intermittency, diffusive effects and spatial variation, and can be used in the simulation environment. However, they do not account for the effect of obstacles on wind field. In the case of dealing with obstacles, open-source, computationally low CFD software like OpenFOAM can be used. After the development of an appropriate plume model, the subsequent task is to choose the suitable mapping algorithm to generate an estimation of the plume from UAV sensors. This will be discussed in the following section.

3. GAS DISTRIBUTION MAPPING

The use of GDM has been extensively reviewed in the scientific literature, with Francis et al. (2022) study being the most comprehensive. Pervious work with a specific focus on methods developed in GDM can be broadly classified into two groups: model-based approaches and model-free approaches.

Model-based gas distribution mapping approaches assume specific gas distribution models and estimate the corresponding parameters based on the measurements. Two model-based methods for mapping a plume using multi-agent systems have been presented in Egorova et al. 2016, Silic and Mohseni 2019. In Egorova et al. 2016, a Luenberger observer is designed for plume dynamics following the 3D advection-diffusion equation.

In lieu of direct measurements of the gradient of the plume, a leader-follower algorithm is adopted to use measurements from multiple spatially-distributed agents to estimate the gradient. The coupling between the control system and estimator navigates the swarm to minimize uncertainty in the estimate of model parameters. In Silic and Mohseni 2019, a parameter estimator is proposed that utilizes online simulation of the plume to inform where to position agents. The performance of their approach is investigated through experiments with three fixed-wing drones, however is limited to a time-invariant model for the plume. These approaches offer several advantages as they require exploring only a small portion of space to create maps quickly. However, they are dependent on well-calibrated gas sensors, an established understanding of the interaction between the sensors and the environment, and often require knowledge about the source intensity. Furthermore, model-based approaches rely on the accuracy of the underlying model, which can be computationally expensive for complex numerical models or based on unrealistic assumptions for simpler analytical models.

Model-free approaches aim to accurately represent measured phenomena and estimate unseen values of at a specific location and time, using a set of collected observations. The majority of publications rely exclusively on spatial information when generating gas distribution models. The GDM approaches that are time-invariant assume that the random parameters being estimated remains constant throughout time. As a result, they model the observed phenomena without taking into account the time of sampling. Some studies have applied methods such as simple averaging of measurements or interpolation of collected data (Ishida, H., et al., 1998), which were employed in experiments conducted over extended periods of time and in confined settings. Hayes et al. 2002, proposed the use of a histogram to represent the gas distribution that involved the averaging of neighboring measurements in the calculation of gas concentration. Lilienthal et al. 2004 utilized a method called Kernel DM, which forecast gas distribution over a grid using a Gaussian kernel. Later, kernel DM+V and Gaussian Process Mixture Model (GPMM) were presented (Lilienthal, A.J., et al., 2009), which estimate both the predictive mean and variance to provide a realist representation of gas concentration fluctuations. Blanco et al. 2013 introduced a Bayesian approach using Kalman Filtering. Reggente et al. 2009 expanded Kernel DM+V to Kernel DM+V+W to incorporate wind information. However, these statistical approaches assume that the gas distribution is generated by a time-invariant random process, which may not accurately represent the current gas distribution in many situations. A few publications have addressed the temporal aspect of GDM; Monroy 2013 developed a time-dependent GDM method using a Gaussian Markov Random Field, while Marjovi et al. 2014 presented a time-dependent approach analogous to Kernel DM+V. Both methods demonstrate better performance than time-invariant GDM approaches in controlled environments. Monroy's method accounts for obstacles while developing a statistical model and estimates gas distribution over a grid. This is achieved through a combination of Euclidean spatial distance and temporal difference weighting of the measurements, which is similar to kernel DM+V. The meta-parameters are defined heuristically with a linear decline in the importance of measurements. Asadi et al. introduced the concept of applying an exponentially reducing recency weight to include time-dependent extrapolation to kernel DM+V. Later they improved their model and introduced TD Kernel DM+V model. They presented two new solutions to combine time-dependency, and evaluated them in both simulated and real-world experiments. Contrary to previous work, in the TD Kernel DM+V, the time-scale factor in the simulation environment is learned along with spatial meta-parameters. The previous work's meta-parameter selection method was sometimes sensitive to initialization values, resulting in overfitting, but these issues have been addressed in the TD Kernel DM+V. We believe that the statistical approaches, particularly TD Kernel DM+V will play an important role in decreasing the dependence of GDM algorithms on time-average dispersion models and they will be the focus of future work. Kernel DM approaches are typically not concerned with the trajectories of robots throughout the search area, often relying on random or pre-defined trajectories. In He et al. 2019, a Gaussian-based kernel method was evaluated experimentally using three multi-rotor drones. He et al.'s method, referred to as Gaussian-plume kernel mapping, was compared to both Kernel DM+V and Kernel DM+V/W, with the latter closely related to Gaussian-plume kernel as both incorporate wind measurements/estimates. In the associated experiments, the search area was partitioned into three, and each agent followed a lawn-mower pattern to cover their individual search areas. Coverage control algorithms for multi-agent networks find locally-optimal spatial distributions for agents, subject to sensing performance and contextual environmental information.

4. UAV SWARM CONTROL SYSTEM

Distributed control of mobile sensor networks provides a variety of desirable properties in real-world environments, such as reduced communication and computational burdens, scalability and robustness (Martinez et al., 2007). Coverage controllers are a class of distributed motion coordination algorithms that is suited to plume estimation.

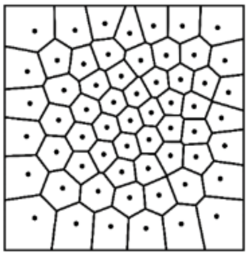

Figure 3. Voronoi partition for 60 agents

The coverage control problem aims to spatially disseminate a network of mobile agents over a desired region subject to an associated cost function. Each agent is equipped with a sensor for measuring a component of the environment (contamination) in assigned sub-regions, and a mapping referred to as a density function assigns a level of importance to each point in the coverage region. The role of the density function depends on the application; in the context of this work, it is used to represent concentration field within the plume. Du et al., 1999 discussed the existence of minimisers of the cost function, which occur when agents are located at the geometric centroid of their coverage sub-regions. Sub-regions are generated by partitioning the space based on the network state, with the optimal partition known as the Voronoi partition (pictured in Figure 3). Intuitively, the Voronoi partition divides the space into sets of points or cells that are closest to a given agent. Voronoi partitions are defined under a particular distance metric; choosing the distance metric that corresponds to the sensor model yields the optimal partition. A centroidal Voronoi tessellation (CVT) refers to configurations where agents are located at the centroid of their respective Voronoi cells, which form the equilibria of the cost function. Cortes et al., 2004 presented a control law for agents with single-integrator dynamics that drives the network to a CVT. Agents compute their control law while only relying on local information from agents with neighbouring Voronoi cells. As agent centroids depend on the density function, the network is attracted to areas of higher density. Figure 4 shows a CVT for 5 agents under a Gaussian density function. In (Kennedy et al., 2019), controllers were presented that are able to track time-varying density functions; an essential characteristic for tracking a dynamic contamination plume.

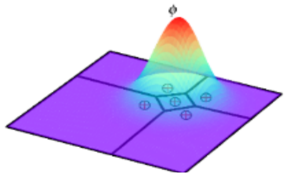

Figure 4. Centroid Voronoi tessellation under Gaussian density function ϕ

Online estimation of the density function has received interest in the literature. Guarantees on convergence of the network and density estimates rely on the representation of the density function. Gaussian regression was used in (Santos et al. 2021) for non-parametric representations, though relies on centralized processing of the estimate. Schwager et al., 2017 developed an estimator for density functions represented by a collection of Gaussian basis functions, however agents are required to transit through the mean of each basis before covering the area to ensure sufficient richness of the estimate. The approach in (Schwager et al., 2017) distributes estimation across the network and guarantee the system converges to a CVT. However, convergence guarantees are only provided for static density functions, which limits their application to plume estimation.

5. CONCLUSION AND FUTURE WORK

This paper presents an overview of recent research on the multiple interrelated components of plume mapping in the simulation environment, including plume simulation model, estimation algorithms and swarm-control strategy. Creating a realistic simulation model will enable us to test and refine the plume mapping system in a controlled environment, before conducting a field test. Therefore, as an initial stage in our future work we need to select an appropriate plume simulation model. As high-fidelity Eulerian models are currently too computationally expensive to use in plume-mapping algorithms, a Lagrangian model may be adequate for the listed purpose. Existing approximated models, such as Filament-based models, are also computationally efficient and can be analysed using Monte Carlo techniques. Once we have selected a suitable plume model, we will compare and evaluate plume-mapping techniques in simulation. The aim is to investigate how effective these techniques are at enabling drones to estimate and map a dynamic contamination plume using coverage controllers. Our study of various plume-mapping algorithms revealed that model-based gas distribution mapping approaches rely on the accuracy of the underlying model. It is impractical to apply complex numerical models that rely on fluid dynamics simulations. Simpler analytical models often based on unrealistic, can only be used in scenarios where these assumptions are valid. Alternatively, most statistical approaches assume that gas dispersion is a time-invariant process. While this assumption holds in some situations, it cannot model well evolving gas plumes. Yet, Time-dependent gas distribution modelling approach such as TD Kernel DM+V enhanced accuracy in forecasting unobserved measurements Therefore, for our use case, TD Kernel DM+V model provides an appropriate representation of the plume and reduces the reliance of GDM algorithms on time-average dispersion models. Coverage control algorithms are well suited for optimal distribution of a dynamic sensor network over a search area. Estimates of the plume can be used to bias the network towards regions with higher concentration, which may lead to more efficient trajectories for agents to sample along in kernel-based estimation approaches. However, as coverage control networks exhibit nonlinear dynamics, the control law and estimator need to be considered together when analysing the stability of the system. A variety of estimation techniques for coverage controllers have been presented in the literature, including both model-based and model-free approaches. Each is limited in their ability to estimate and track dynamic plumes in a distributed setting. Future work involves providing stability guarantees for time-varying coverage controllers using both kernel-based estimators and Luenberger observers for time-varying plume representations, and experimental validation of the listed approaches. Future work involves expanding a coverage control system from 2D to 3D, providing stability guarantees for time-varying coverage controllers and the selected plume-mapping algorithm in preparation for real-world experiments. Outdoor field experiments with a swarm of UAVs will be conducted to verify that the proposed solution can map plumes with a suitable sensor suite.

REFERENCES

Belanger, J.H. and Willis, M.A., 1996. Adaptive Control of Odor-Guided Locomotion: Behavioral Flexibility as an Antidote to Environmental Unpredictability1. Adaptive Behavior, 4(3-4), pp.217-253.

Blanco, et al., 2013, March. A kalman filter based approach to probabilistic gas distribution mapping. In Proceedings of the 28th Annual ACM Symposium on Applied Computing (pp. 217-222).

Chen, X. and J. Huang, 2019. Odor source localization algorithms on mobile robots: A review and future outlook. Robotics and Autonomous Systems 112: 123-136.

Cortés, J., et al., 2004. Coverage Control for Mobile Sensing Networks. IEEE Transactions on Robotics and Automation, 20(2), 243–255.

Du, Q., et al., 1999. Centroidal Voronoi tessellations, SIAM Review, 41(4), 637–676.

Egorova, T., et al., 2016, December. Plume estimation using static and dynamic formations of unmanned aerial vehicles. In 2016 IEEE 55th Conference on Decision and Control (CDC) (pp. 2270-2275). IEEE.

Farrell, J.A., et al., 2002. Filament-based atmospheric dispersion model to achieve short time-scale structure of odor plumes. Environmental fluid mechanics, 2, pp.143-169.

Francis, A., Li, S., Griffiths, C. and Sienz, J., 2022. Gas source localization and mapping with mobile robots: A review. Journal of Field Robotics, 39(8), pp.1341-1373.

Gongora, A., et al., 2020. Joint estimation of gas and wind maps for fast-response applications. Applied Mathematical Modelling 87: 655-674.

Hayes, A.T., et al., 2002. Distributed odor source localization. IEEE Sensors Journal, 2(3), pp.260-271.

He, X., et al., 2019, October. Gaussian-based kernel for multi-agent aerial chemical-plume mapping. In Dynamic Systems and Control Conference (Vol. 59162, p. V003T21A004). ASME.

Ishida, H., et al., 2001. Plume-tracking robots: A new application of chemical sensors. The Biological Bulletin 200(2): 222-226.

Ishida, H., et al., 2012. Chemical sensing in robotic applications. IEEE Sensors Journal 12(11): 3163-3173.

Kennedy, J., et al., 2019. Generalized Coverage Control for Time-Varying Density Functions. 2019 18th European Control Conference (ECC), 71–76.

Marjovi, A. and L. Marques, 2014. Multi-robot odor distribution mapping in realistic time-variant conditions. 2014 IEEE International Conference on Robotics and Automation (ICRA), IEEE.

Martínez, S., et al., 2007. Motion coordination with distributed information. IEEE Control Systems Magazine, 27(4), 75–88.

Monroy, J., 2013. Advances in gas sensing and mapping for mobile robotics.

Leelőssy, Á., et al., 2018. A review of numerical models to predict the atmospheric dispersion of radionuclides. Journal of environmental radioactivity 182: 20-33.

Leong, A.S. et al., 2022. Field estimation using binary measurements. Signal Processing, 194, p.108430.

Lilienthal, A. J., et al., 2006. Airborne chemical sensing with mobile robots. Sensors 6(11): 1616-1678.

Lilienthal, A.J., et al., 2009, October. A statistical approach to gas distribution modelling with mobile robots-the kernel DM+V algorithm. In 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems (pp. 570-576). IEEE.

Lochmatter, T. and Martinoli, A., 2009. Understanding the potential impact of multiple robots in odor source localization. Distributed Autonomous Robotic Systems 8, pp.239-250

Reggente, M. and Lilienthal, A.J., 2009, October. Using local wind information for gas distribution mapping in outdoor environments with a mobile robot. In SENSORS, 2009 IEEE (pp. 1715-1720). IEEE.

Rosser, K., et al., 2015. Autonomous chemical vapour detection by micro-UAV. Remote Sensing 7(12): 16865-16882.

Salmon, P., 2015. Non-linear pattern formation in bone growth. Frontiers in Endocrinology, 5, p.239.

Santos, M., et al., 2021. Multi-robot Learning and Coverage of Unknown Spatial Fields. 2021 International Symposium on Multi-Robot and Multi-Agent Systems, MRS 2021, 137–145.

Schwager, M., et al., 2017. Robust Adaptive Coverage Control for Robotic Sensor Networks. IEEE Transactions on Control of Network Systems, 4(3), 462–476.

Silic, M. and Mohseni, K., 2019. Field deployment of a plume monitoring UAV flock. IEEE Robotics and Automation Letters, 4(2), pp.769-775.

Sutton, O.G.,1953, Micrometerology, McGraw-Hill, New York.