

Optimal decision making and control with uncertain events, uncertain physics, or both

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Abstract: Systems can be subject to exogenous uncertainty, that is, uncertainty about the future stimuli on the system (e.g., the time and location of bushfires occurring in a region) and/or the parameters of important influences that lie at the boundary of the system (e.g., the time-varying price of grid-sourced electricity). All real systems have exogenous uncertainty but for some systems a deterministic model can be sufficient for good decision-making about system design and/or operation. The Operations Research (OR) literature has tended to favour deterministic models but there is much literature associated with optimisation under uncertainty. For the most part this uncertainty is wholly exogenous in the literature.

In physical science and engineering the endogenous uncertainty and unpredictability of systems is ever-present. Knowledge of the physics which underpins the system's behaviour is almost never complete enough to enable high-accuracy prediction. The predictions which are achievable are often semi-empirical in nature — being partly physics informed (model driven) and using observed experimental data to fill knowledge gaps (data driven). This means that the response of a system to a control action might only be quite imprecisely known even if exogenous influences are kept entirely at bay. Endogenous uncertainty is less commonly tackled in the OR literature. This is partly for practicality because combinatorial optimisation problems are quite difficult enough. This is also partly a result of context. With regards to the latter, modelling for planning and scheduling problems, for example, does not benefit much from questioning the accuracy of the underlying physics. In these cases it is considered enough to restrict and/or refer the uncertainty and unpredictability to the exogenous influences (such as traffic congestion in transportation, or the arrival of new tasks at a production system).

There are favourable circumstances where methods for optimisation under uncertainty do not involve generating and fitting functions to somewhat large amounts of data. For example, if uncertainties are about discrete event realizations and are few in number, then a scenario tree can be enumerated and a problem can be solved using multi-stage stochastic programming. As problems get more complex, methods for optimisation under uncertainty can be said to become data-driven approaches, as exemplified when estimating the cost-to-go function in approximate dynamic programming using a Least Squares Monte Carlo method.

Our motivation here is to explore the notion that data-driven physics representations and data-driven stochastic optimisation might not need to be treated as two compartmentalized tasks. Might we be able to undertake approximate dynamic programming for process control *and* physics-based model fitting for process prediction simultaneously? How might this work? What then might the relationship become between physical experimentation, “digital twins”, and computations for stochastic optimisation?

In exploring this notion we walk through a series of examples where complex uncertain systems and combinatorial optimisation have already been simultaneously addressed by the authors and their collaborators: (i) multi-period sizing of heating, cooling and electricity generation equipment in houses; (ii) the dynamic allocation of aerial firefighting resources to bushfires; (iii) designing roads to minimise environmental disruption; and (iv) simultaneous electric vehicle charging and electricity market participation. These examples enable us to build a taxonomy of different ways in which combinatorial optimisation and stochastic simulation ideas can be combined to solve decision-making and control problems. It also helps us frame and describe the challenges and possibilities when considering the dual data-driven physics and data-driven stochastic optimisation notion.

Keywords: *Optimisation, decision making under uncertainty, Monte Carlo methods, digital twins*