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Optimal historical volatility for the Dow-Jones industrial average and its application

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Abstract: We investigate the daily share prices of Dow-Jones industrial average to identify jump times using a jump diffusion model, which consists of the Black-Scholes model with stochastic volatility and a compound Poisson process. We estimate optimal values of stochastic volatility of the Dow-Jones industrial average during the period 1985 - 2015. From the results, we conclude that the threshold level of the volatility for some big changes like the 2008 financial crisis is around 0.01.

Keywords: *Jump diffusion model, Dow-Jones industrial average, stochastic volatility, historical volatility*

1 INTRODUCTION

In this paper we show an algorithm to find an optimal historical volatility from daily share prices of stock indexes according to Kanagawa, Namekawa and Shinkai (2019) and Kanagawa and Shieh (2023). From the results, we see that the threshold level of the historical volatility of the Dow-Jones industrial average for some big changes like the Lehman shock is 0.01.

2 MODELING OF SHARE PRICES

The following equation is a jump diffusion model of share prices proposed by Ball and Torous (1983), (1985), Cont and Tankov (2008), Iino and Ozaki (1999), Kou (2002), etc.;

$$dS\left(t\right) = \mu S\left(t\right)dt + \sigma S\left(t\right)dB\left(t\right) + S\left(t\right)dZ\left(t\right), \quad 0 \le t \le T \tag{1}$$

for small T, where $S\left(t\right)$ is a share price, $B\left(t\right)$ is a standard Brownian motion, μ is a trend parameter, $\sigma_t > 0$ is a volatility and $Z\left(t\right)$ is a compound Poisson process. Furthermore the independence of $B\left(t\right)$ and $Z\left(t\right)$ is assumed. Since the volatility σ and the trend μ change in a long period, recently the model (1) was improved as the form

$$dS\left(t\right) = \mu_t S\left(t\right)dt + \sigma_t S\left(t\right)dB\left(t\right) + S\left(t\right)dZ\left(t\right), \quad 0 \le t \le T, \tag{2}$$

where μ_t is a stochastic trend and σ_t is a stochastic volatility.

On the other hand, in Ishida and Kanagawa (2015) and Kanagawa and Shieh (2019) we investigated the daily share prices of Nikkei 225 indexes to estimate the jump times of large size jumps. Since the usual model (2) cannot fit our results, we would like to propose the following new jump diffusion model;

$$dS\left(t\right) = \mu_t S\left(t\right)dt + \sigma_t S\left(t\right)dB\left(t\right) + \sigma_t S\left(t\right)dZ\left(t\right), \quad 0 \le t \le T. \tag{3}$$

In the new model (3) the intensity $\lambda > 0$ of the compound Poisson process $Z\left(t\right)$ is a constant in the whole period $[0, T]$. Although the risk of the share price $S\left(t\right)$ changes very frequently in $[0, T]$, we would like to show that the cause of the risk change mainly due to σ_t in (2) but not λ from the obtained data.

3 ESTIMATION OF THE TREND μ_t AND THE VOLATILITY $\sigma_t > 0$

We estimate μ_t and $\sigma_t > 0$ from the Nikkei 225 indexes data. Let

$$r\left(k\right) = \log\frac{S\left(k+1\right)}{S\left(k\right)} \simeq \frac{S\left(k+1\right) - S\left(k\right)}{S\left(k\right)} \tag{4}$$

be the return of the kth day and put

$$s_t = \sqrt{\frac{1}{\ell - 1}\sum_{k=1}^{\ell}\left(r\left(t-k\right) - \bar{r}\right)^2}, \quad \bar{r} = \frac{1}{\ell}\sum_{k=1}^{\ell} r\left(t-k\right). \tag{5}$$

s_t is the historical volatility estimating $\sigma_t > 0$ using share prices of ℓ days. Furthermore let

$$\bar{\mu}\left(t\right) = \log\frac{S\left(t\right)}{S\left(t-\ell\right)} \tag{6}$$

be an estimator of μ_t.

Standardize $r(t)$;

$$R\left(t\right) = \frac{r\left(t\right) - \bar{r}\left(t\right)}{s_t} \tag{7}$$

4 IDENTIFICATION APPROACH OF OPTIMAL HISTORICAL VOLATILITY

Since the counting process $N\left(t\right)$ obeys Poisson distribution

$$P\left\{N\left(t\right) = k\right\} = e^{-\lambda t}\frac{\left(\lambda t\right)^k}{k!}, \quad k = 0, 1, 2, \cdots, \tag{8}$$

we estimate the intensity λ by the following steps.

We define an *unit period* with q days and divide 7447 days of 30 years from 1986 to 2015 into

$$L = \left[\frac{7450}{q} \right] \tag{9}$$

periods.

We count the number of jumps observed in each period which means a sample of $N(1)$.

We identify jump times of $R(t)$ for large size jumps in each period by the following way.

1st step : Put a threshold level $\alpha > 0$ which controls the jump size.

2nd step: If $|R(k)| \geq \alpha$ in the jth period, then we consider that the kth day is a jump time. Let m_j be the number of all jumps larger than α in the $j$the period. $m_j, j = 1, 2, ..., L$ are samples of $N(1)$.

3rd step: For the samples $m_j, j = 1, 2, ..., L$, put

$$\lambda_\alpha = \frac{1}{L}(m_1 + m_2 + \cdots + m_L), \tag{10}$$

which is an estimator of intensity of Poisson distribution.

4th step: Let n_k be the number of periods having k jumps in a period. From the definition of n_k

$$n_0 + n_1 + \cdots + n_K = L,$$

where K is the maximum of number of jumps in a period.

5th step: Calculate the following estimator χ^2_α for the test of goodness of fit to Poisson distribution with the intensity λ_α for the samples $n_0, n_1, \cdots, n_K$.

$$\chi^2_\alpha = \sum_{k=0}^{K+1} \frac{(n_k - kp_k)^2}{kp_k}, \tag{11}$$

where

$$p_k = e^{-\lambda_\alpha t} \frac{(\lambda_\alpha t)^k}{k!}, \quad k = 0, 1, 2, \cdots, K$$

and

$$p_{K+1} = 1 - \sum_{k=1}^{K} e^{-\lambda_\alpha t} \frac{(\lambda_\alpha t)^k}{k!}.$$

6th step: Compare χ^2_α for all combinations of number of days in a unit period q and number of days to observe the historical volatility with ℓ. Calculate χ^2_α for each $\alpha = 1.0, 1.1, \cdots, 2.6$.

7th step: Find the historical volatility s_t which is determined by a unit period q and number of days ℓ minimizing χ^2_α. If the minimum χ^2_α is small enough, we conclude that the historical volatility s_t is optimal since the above large returns $r(t)$ are Poisson-distributed and they are generated by the compound Poisson process $Z(t)$.

5 Optimal historical volatility for the the Dow-Jones industrial average

We would like to explain Tables 1 and 2 obtained from Kanagawa and Shieh (2023).

From Table 1, we find that χ^2_β takes the minimum value 0.89 when $\beta = 2.4$, $q = 17$ for 30 years observation period (18, April, 1985 $\sim$ 22, April, 2015). Furthermore, in Table 2, we can see that the difference between

n_k (the number of periods having k jumps in a period) and its theoritical value for each $1 \leq k \leq 5$ for the Poisson distribution is very small in the case $\chi^2_\beta = 0.89$.

From the above procedure, we can see that the optimal historical volatility s_t is obtained when $q = 17$.

Fig.1 shows the historical volatilities of the Dow-Jones industrial average in the period 18, April,1985 $\sim$ 22, April, 2015. On the other hand, Fig.2 is a part of Fig.1 picked up from 1, July, 2008 $\sim$ 31, October, 2008 around the 2008 financial crisis.

From Fig.1 and Fig.2, we conclude that the threshold level of the volatility for some big changes like the 2008 financial crisis is around 0.01.

Table 1. Dow-Jones, χ^2_β ($\beta = 2.4$) for each ℓ and q (18, April, 1985 $\sim$ 22, April, 2015)

ℓ							
32	11.30	12.85	8.92	7.18	5.27		
31	5.46	3.72	4.54	3.84	2.92		
30	7.61	4.06	3.30	1.53	2.02		
29	5.36	1.78	2.20	2.52	0.89		
28	5.68	4.10	2.43	2.92	3.25		
27	12.76	6.93	8.47	5.11	4.12		
	13	14	15	16	17	q	

Table 2. Dow-Jones, $\ell = 29, q = 17, \beta = 2.4, \lambda_\beta = 0.81, \chi^2_\beta = 0.89$ (18, April, 1985 $\sim$ 22, April, 2015)

Number of jumps (k)	n_k	Lp_k
0	112	110.04
1	95	89.41
2	36	36.32
3	11	9.83
4 (K)	2	1.99
5 ($K+1$)	0	0.32

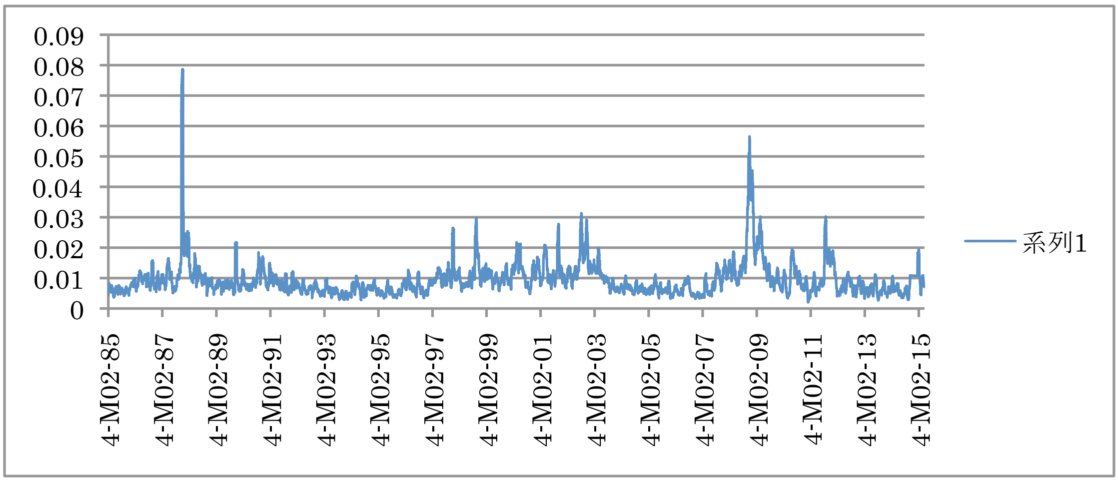

Figure 1. s_t of Dow-Jones (18, April,1985 $\sim$ 22, April, 2015)

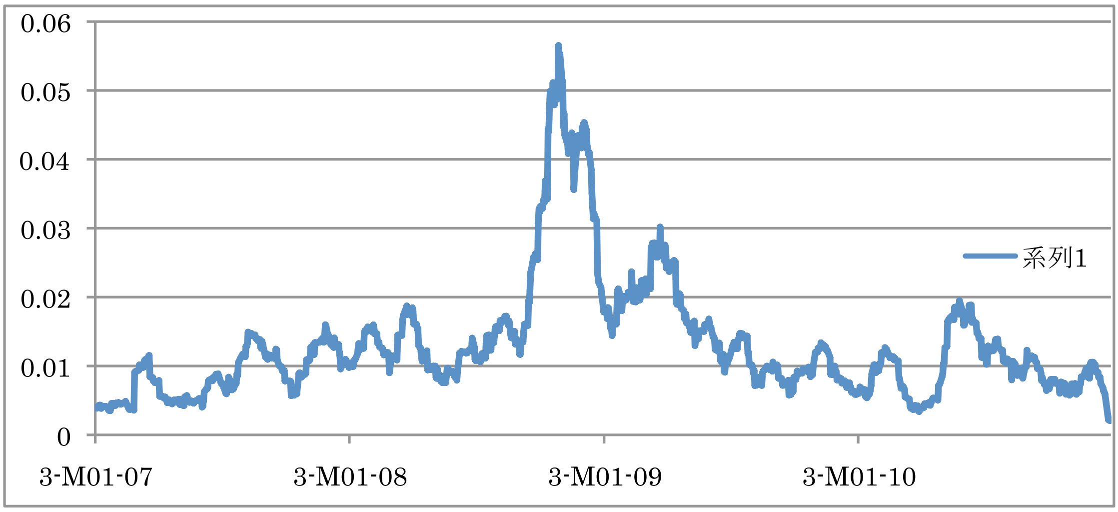

Figure 2. s_t of Dow-Jones at the 2008 financial crisis (1, July, 2008 $\sim$ 31, October, 2008)

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