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An optimised strategy for using cooperating missiles for missile defence

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Abstract: This paper describes an optimal strategy for defending against an attacking missile with a team of cooperating defending missiles. The motivation is to allow us to compare a single high-cost defending missile to a team of low-cost defending missiles.

The strategy predicts the possible paths of the attacking missile from a known initial state to a known target, then determines two regions: the region the attacking missile will be in when the defending missiles start seeking the attacking missile, and the region the attacking missile will be in when it can be intercepted.

The selection of aim points that maximise the defending missile team's coverage of these two regions can be formulated as a multi-dimensional unconstrained non-linear optimisation problem.

The optimisation determines whether it is better to have thin coverage of the entire seek and intercept regions or concentrated coverage on parts of the seek and intercept regions to maximise the probability of stopping the attacking missile. In general, the more paths that each defending missile can see and hit, the greater the probability of stopping the attacking missile.

The optimisation problem is solved using the Nelder-Mead method and an example is given to demonstrate the effectiveness of the method.

Keywords: *Cooperative control, team defence, air missile defence systems, unconstrained non-linear programming*

1 Introduction

The aim of this work was to determine whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve performance and cost-effectiveness of missile defence systems. Figure 1 shows a missile defence system defending against a high-performance attacking missile. The target of the attacking missile is known, but the trajectory that the attacking missile will take is unknown. The defenders must prevent the attacker from entering the *keep-out zone* of the protected target.

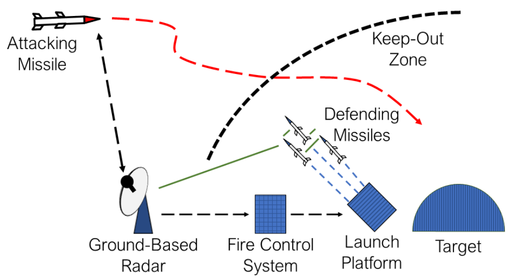


Figure 1. Components of a missile defence system

There are several events that occur during a missile engagement:

1. The attacker is detected by the ground-based radar. The ground-based radar will provide position and velocity measurements to the fire control system and defenders before and after launch.
2. The fire control system will determine how many defenders to launch, when to launch them, and where each defender should fly to before it turns on its seeker. As the defenders fly out towards the attacker, measurements from the ground-based radar will be used to refine the possible paths of the attacker, which in turn will be used to refine the points that the defenders are flying towards.
3. Each defender will be equipped with a *seeker* that can detect the attacker. The defender seekers will provide continuous and more accurate measurements of the attacker position and velocity compared to the ground-based radar. The defenders will turn on their on-board seekers at the time when the attacker is predicted to come within the seeking range of the defenders. By this time, the defender positions and directions should be organised such that they will be able to see and intercept all possible paths of the attacker.
4. Once the attacker is detected by at least one defender, the team of defenders will coordinate to *track* and *intercept* the attacker with as many defenders as possible.

The objective is to maximise the probability of stopping the attacker. The problem is formulated using simple models (Section 2). Even though the models are simple, they can lead to useful insights and conclusions which can be used in more detailed analysis, perhaps making the detailed analysis phase easier and shorter. This paper describes a cooperative control strategy based on predicting the possible paths of the attacker from a known initial state to a known target (Section 2.1), then defining the region that the attacker will be in when the defenders start seeking, when it can be intercepted (Section 2.2). These two regions are used to determine points for the defenders to aim towards before operating their on-board seekers. The selection of *aim points* for the defenders to fly towards such that the defenders are able to maximise their coverage of the seek and intercept regions is a multi-dimensional unconstrained non-linear optimisation problem (Section 2.3). The optimisation determines whether it is better to have thin coverage of the entire seek and intercept regions or concentrated coverage of parts of the seek and intercept regions to maximise the probability of intercepting the attacker. The Nelder-Mead method [Nelder and Mead, 1965] is used to solve the optimisation problem, though any similar method could be used. An example is given showing the simulation of the cooperative control strategy from first radar detection to when the defenders fly-out and turn-on their on-board seekers (Section 3).

2 Problem Formulation

This section develops an optimal control strategy for a team of defenders,and extends an earlier 2D version of this work [Kapsis et al., 2021]. Consider a simplified 3D problem where curvature of the Earth is ignored. We want to defend against a single attacking missile that is assumed to fly towards a stationary target at position $(0, 0, 0)$. The coordinate system has x pointing east, y pointing north and z pointing up. Suppose the attacker is detected at time t_0 by a ground-based radar, and is at position P_0 with velocity $\mathbf{v}_\mathrm{a}$. We have up to M defenders that can cooperatively seek and intercept the attacker. The defenders must prevent the attacker from entering the *keep-out* zone of the protected target. We assume that the target is stationary, but it would be straightforward to extend the methods we develop to protect a moving asset, or a group of assets. Our method to defend against the attacker requires the following steps:

1. Given a radar measurement of the position and velocity of the attacker, predict possible paths of the attacker towards the stationary target. The attacker is able to perform aerodynamic manoeuvres and its future path towards the target is unknown. In the situation where there could be multiple possible targets, then predicted attacking paths should be generated towards each target.
2. Define a *seek region* comprising points on the predicted attacking paths where the attacker could be at a calculated *seek time*, and define an *intercept region* comprising points on the predicted attacking paths where the attacker could be at an approximated *intercept time*. Then define an *aim point* for each defender to fly towards so the defenders will be able to effectively seek and intercept the attacker.

Prior to launching the defenders, and during their fly-out phase, the ground-based radar will provide periodic updates on the attacker's position and velocity. At every radar update, the predicted attacking paths, intercept time, seek time, seek region, intercept region, aim points and defending paths will change. The challenge is to find aim points that will minimise the probability of missing the attacker and ensure the defending paths remain feasible.

2.1 Predicting attacking missile paths

An attacking path is a smooth curve that terminates at the stationary target location. Missiles are limited in their ability to manoeuvre by aerodynamic and structural constraints [Raymer, 1992], which define the maximum curvature of the path. Kapsis et al. [2021] generated Bezier curves to represent possible attacking paths, as they are smooth, easy to generate and allow the missile to manoeuvre while respecting maximum manoeuvrability constraints. Increasing the number of predicted attacking paths will better represent the manoeuvre region of the attacker.

Although not every possible path or type of manoeuvre can be represented, this method appears to give a realistic envelope of paths and can be used to estimate the density of possible paths in any region. Figure 2 shows 500 sample predicted missile paths (orange lines) for an attacker detected at position $\mathbf{P}_0 = (30\,\text{km}, 0\,\text{km}, 7\,\text{km})$ (red point) with velocity $\mathbf{v}_\text{a} = (-0.6\,\text{kms}^{-1}, 0\,\text{kms}^{-1}, 0\,\text{kms}^{-1})$. We assume that the attacker will travel at a constant speed throughout its flight. Paths with curvature requiring a lateral acceleration greater than $5g$, where g is gravitational acceleration $(9.81\,\text{ms}^{-2})$, have been discarded. The actual path of the attacker (red line) is unknown to the defending team.

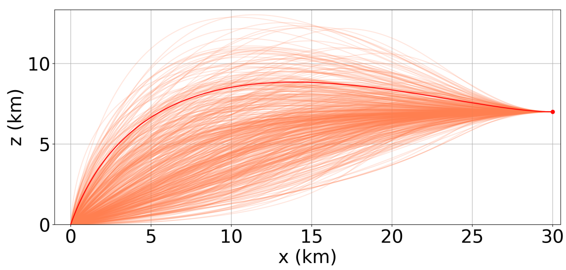


Figure 2. 500 possible attacking missile paths

2.2 Intercept and seek regions

Each defender j will be launched at some time $t_{\text{L}j} > t_0$ and will fly towards an aim point where it will turn on its seeker to seek and then attempt to intercept the attacker. An aim point specifies the time, position and direction $(t_{\text{da}j}, \mathbf{P}_{\text{da}j}, \mathbf{d}_{\text{da}j})$ of the defender j. The 'da' in the subscript indicates 'defender at aim point'. Assuming defender j reaches its aim point at time $t_{\text{da}j}$, it must then manoeuvre to intercept the attacker at a chosen intercept time. The intercept region comprises points $\mathbf{P}_i(t_\text{i})$ that occur along each predicted attacking path $\mathbf{P}_i$ at some chosen intercept time t_i, as shown by the red points in Figure 3b. When choosing the intercept time, it is important to consider the target's keep-out zone as this will be the defender's minimum intercept range r_i from the target. By waiting for the attacker to get closer to the target, the remaining time for the attacker to manoeuvre towards its target reduces and the maneuverability region of the attacker becomes smaller, which makes the attacker easier for the defenders to intercept. Therefore, it is desirable for the defenders to intercept the attacker close to the keep-out zone.

The target intercept time is estimated by first approximating the attacking path as a straight line between its position $\mathbf{P}_0$ and the position of the target $\mathbf{P}_3$, and then calculating the time at which the attacker will reach the keep-out zone. Thus, the target intercept time is

$$t_\text{i} = t_0 + \frac{|\mathbf{P}_3 - \mathbf{P}_0| - r_\text{i}}{|\mathbf{v}_\text{a}|}. \tag{1}$$

Equation 1 is the earliest time the attacker will reach the intercept region. This is an underestimation of the actual intercept time which means that the defenders will be launched earlier than needed. However, it is likely that the defenders will be delayed as their paths will change with every radar update. Therefore underestimating the intercept time is advantageous as it is less likely that the attacker will arrive at the intercept region before the defenders. This has the desired effect of causing the defenders to intercept the attacker near, but outside, the keep-out zone.

Each defender will have a seeker with a detection range of r_s, that is used to measure the position of the attacker and steer the defenders towards the attacker. The attacker cannot be intercepted unless it is seen by at least one defender. Each defender will fly towards an aim point which specifies the time, position and direction for the defender seeker to first detect the attacker. The time which the defenders arrive at their aim point is the seek time. This is the earliest time that any defender can detect the attacker from the aim point. The seek region is where the attacker could first be detected by any defender's seeker from the aim point. Figure 3a shows the seek region (blue points) which comprise points $\mathbf{P}_i(t_\text{s})$ that occur along each predicted attacking path $\mathbf{P}_i$ at a calculated seek time t_s. The attacker will reach the seek region before it reaches the intercept region, so the seek time occurs before the intercept time where $t_\text{s} < t_\text{i}$.

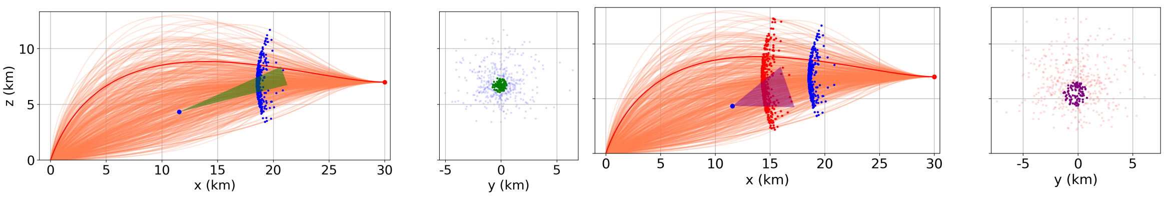
The seek time is calculated by first assuming that the attacker and defender are flying towards each other along a straight line at constant speeds and will intercept at time t_i. If the attacker is traveling at constant speed $|\mathbf{v}_\text{a}|$ and the defender at constant speed $|\mathbf{v}_\text{d}|$ then the closing speed is $|\mathbf{v}_\text{a}| + |\mathbf{v}_\text{d}|$. The defender can detect the attacker when the two are at distance r_s apart. Thus the time at which seeking should begin is

$$t_\text{s} = t_\text{i} - \frac{r_\text{s}}{|\mathbf{v}_\text{a}| + |\mathbf{v}_\text{d}|}. \tag{2}$$

Equation 2 underestimates the seek time which means that the defenders will arrive at their aim points and turn on their seekers earlier than needed to intercept the attacker outside the keep-out zone. This is a strategic decision to ensure that the attacker does not arrive at the seek region before the defenders turn on their seeker, while keeping calculations simple.

The detection region of a defender is modelled as a conical sector of a sphere with the center point at the aim point position (blue point), as shown by the green triangle in the left image of Figure 3a. The rightmost image in Figure 3a shows the seek points in the yz plane that the defender can (green points) and cannot (pale blue points) see. Once an attacker path is seen by one defender, it is considered seen by all defenders. The more attacker paths that the defenders can see, the more likely that the attacker will be detected. Once one defender detects the attacker, then all the defenders will manoeuvre to intercept the attacker.

Assuming defender j reaches its aim point at time t_s, it must then manoeuvre to intercept the attacker at the intercept time t_i. A defender will be able to intercept a predicted attacking path if the predicted intercept point along that path is within the defender's manoeuvre region, shown by the purple triangle in the left image of Figure 3b. The manoeuvre region of each defender is approximated as a conical sector of a sphere with the center point at the aim point position $\mathbf{P}_{\text{da}j}$. The half-angle of the manoeuvre cone is limited by the minimum turning radius of defender j and the time that defender j has to reach the intercept region. The rightmost image in Figure 3b shows the intercept points in the yz plane that the defender can (purple points) and cannot (pale red points) hit.



(a) Seek region (blue points) and defender's detection region (green).

(b) Intercept region (red points) and defender's manoeuvre region (purple).

Figure 3. Defending missile at $\mathbf{P}_{\text{a}j} = (11.547\text{ km}, -0.284\text{ km}, 4.332\text{ km})$ with a maximum lateral acceleration of $4g$ and constant speed of 0.8 km s^{-1}. with a seeker view angle of $\pm 5°$ and a detection range of 10 km.

2.3 Optimising aim points

Suppose there are M defenders. Each defender j will fly towards its aim point $(t_{\text{da}j}, \mathbf{P}_{\text{da}j}, \mathbf{d}_{\text{da}j})$. The objective is to choose aim points that will maximise the likelihood of hitting (and stopping) the attacker. It is convenient to write this objective as minimising the probability of missing the attacker with every defender. Suppose there are N predicted attacking paths. The team of defenders can intercept the predicted attacking path i only if path i is seen by at least one defender. The probability of the team of defenders missing path i is:

$$m_i = \begin{cases} 1 & \text{if path } i \text{ cannot be seen} \\ f^{\mu_i} & \text{if path } i \text{ can be seen} \end{cases}$$

where f is the probability of an individual defender failing to stop the attacker even though it can manoeuvre to intercept path i, and μ_i is the number of defenders that can feasibly intercept path i. The probability of the team of defenders missing the attacker is

$$P_{\text{miss}} = \sum_{i=1}^{\text{N}} p_i m_i$$

where p_i is the probability that the attacker will take the predicted path i, and where $\sum_i^N p_i = 1$. A similar probability of miss has been described by Azzollini et al. [2021].

The manoeuvre region of the attacking missile is not uniformly distributed. Furthermore, an attempted intercept on a particular path may fail, so the probability of missing the attacker on a particular path decreases with the number of attempted intercepts. The optimisation needs to determine whether it is beneficial to thinly cover the entire seek and intercept regions or to concentrate coverage in dense areas of the seek and intercept regions to minimise the probability of missing the attacker. In general, the more paths that each defender can see and hit, the greater the probability of hitting (and stopping) the attacker.

After launching the defenders, periodic updates of the attacker's position and direction are received from the ground-based radar. At every update, new aim points with new defending paths are calculated. Defenders have restricted manoeuvrability (turning rate), therefore the new aim points need to be positioned such that each defender can correct its path towards its new aim point. Suppose defender j has direction vector $\mathbf{d}_{\text{d}jk}$ at the k^{th} radar update where $k \geq 0$ and the updated aim point vector is $\mathbf{d}_{\text{da}jk}$. We can approximate the angle that the defender will have to turn through by the angles:

- $|\theta_{\text{d}}|$ between the current direction and the line from the current location to the aim point location, and
- $|\theta_{\text{da}}|$ between the line from the current location to the aim point location and the required direction at the aim point.

The direction vectors (red lines) and the angles that the defender will turn through are shown in Figure 4. In this scenario, the defender will have to turn right away from its current path then left to achieve the aim direction to ensure that it's on-board seeker is pointed towards the attacker when it enters the seek region. The total change in heading for the two direction changes will be $\Theta_j = |\theta_{\text{d}j}| + |\theta_{\text{da}j}|$.

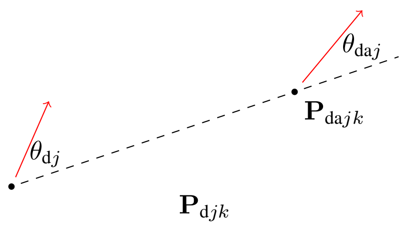


Figure 4. Angles a defender will turn through.

We could introduce a hard constraint $\Theta \leq \Theta_{\max}$, but this would mean introducing Lagrange multipliers and Karush-Kuhn-Tucker conditions to solve a non-linear constrained optimisation problem [Beavis and Dobbs, 1990]. Instead, since smaller angle changes are more desirable, we add a simple penalty function to our objective function to give the new optimisation problem: **find aim points** $(t_{\text{da}j}, \mathbf{P}_{\text{da}j}, \mathbf{d}_{\text{da}j})$ **to minimise**

$$\zeta = P_{\text{miss}} + qP_{\Theta} \tag{3}$$

where the penalty for turns is given by the mean of the angles squared

$$P_{\Theta} = \frac{\sum_{j=1}^{M} \Theta_j^2}{M}$$

and where q is the penalty weight [Bazaraa et al., 2006]. As the defenders approach the intercept region, it becomes less feasible for them to achieve high angle changes to their aim points because of the limited time and space available for them to manoeuvre. We increase the penalty weight at every radar update to avoid large angle changes as the defenders get closer to the aim points. We use $q(k+1) = q(t_0)(\log(k+1)+1)$.

Other penalty functions P_Θ were considered. We found that the mean of the angles squared provided a lower probability of miss on average compared to the other penalty functions. We solve the unconstrained non-linear optimisation problem using the Nelder-Mead method [Nelder and Mead, 1965], which is a direct search method commonly used to solve unconstrained multi-dimensional non-linear problems as it does not require derivatives [Gao and Han, 2010] and is easy to implement. Alternative numerical methods could be used.

3 Example

We illustrate the feasibility of our approach with an example using ten defenders from Kapsis et al. [2023]. Figure 5 shows how the seek region, intercept region and aim points evolve as information about the attacker is updated by the ground-based radar every 4 seconds. The attacker is initially detected at time $t_0 = 0$ at position $\mathbf{P}_0 = (30\,\text{km}, 0\,\text{m}, 7\,\text{km})$ (red dot) with velocity $\mathbf{v}_\text{a} = (-0.6\,\text{kms}^{-1}, 0\,\text{kms}^{-1}, 0\,\text{kms}^{-1})$. The attacker and defenders are travelling at constant speeds $|\mathbf{v}_\text{a}| = 0.6\,\text{kms}^{-1}$ and $|\mathbf{v}_\text{d}| = 0.8\,\text{kms}^{-1}$ respectively. Aim points are calculated for ten defenders. Each defender has a probability of failing to intercept $f = 0.1$ and a maximum lateral acceleration of $4g$. The path for each defender is formed from a current position and direction to its aim point using a geometric Hermite curve [Yong and Cheng, 2004]. The launch direction for each defender is set to intercept the aim direction at a point one tenth of the way between x_d and x_da. This launch gives good results as it ensures that the defender's fly-out path has low curvature, and that future positions and directions along the fly-out path will be suitable for generating future paths when the position of the attacker is updated from ground-based radar measurements. The objective function has an initial penalty weight $q(t_0) = 0.16$. The probability of miss when the attacker was initially detected was $P_\text{miss}(t_0) = 0.3203$, and reduced to $P_\text{miss}(t_3) = 0.0102$ after three radar updates. If the team of defenders were able to cover all seek and intercept points then the probability of miss could be minimised to $P_\text{miss} = 10^{-10}$. The optimisation took 3–4.5 seconds. This is slow for real-time control, but sufficient to evaluate the viability of cooperative defenders.

4 Conclusions and Future Work

This paper developed and illustrated a method for constructing a cooperative control strategy for a team of defenders to cover a seek and intercept region that takes into account the manoeuvrability range of an attacker. The method predicts a set of possible paths that an attacker could take, then constructs defending paths that will see and intercept as many of these paths as possible to minimise the probability of missing the attacker. The problem of choosing trajectories for the defenders can be formulated as an unconstrained non-linear programming problem, where the objective is to choose aim points for the defending missiles to maximise their coverage of the seek and intercept regions. The more paths that the defenders are able to see and hit, the greater the probability of intercepting the attacking missile. A challenge was to ensure that the defending missile paths remained feasible after launch. Introducing a term to the objective function that penalises large steering angles achieved this, whilst keeping the optimisation problem unconstrained so that it could be solved using numerical methods such as the Nelder-Mead method [Nelder and Mead, 1965]. Further work could include comparing the performance of the Nelder-Mead method to other numerical methods. The demonstration of our cooperative strategy could be enhanced by incorporating higher fidelity models of the missile defence system. The effectiveness of our cooperative strategy could be analysed by comparing the performance to other cooperative strategies such as the staggered launch strategy [Shaferman and Oshman, 2016] or the predictive regret-matching algorithm [Rajagopalan et al., 2019].

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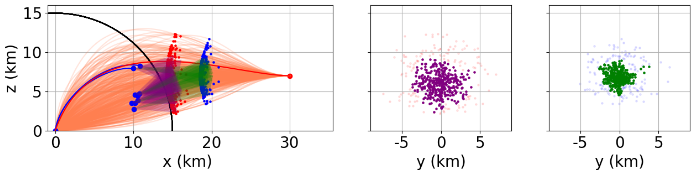
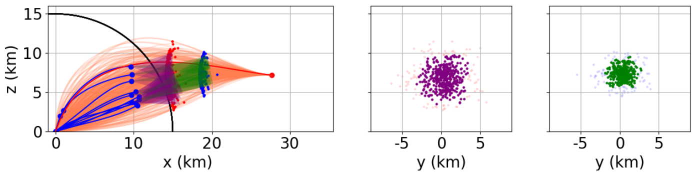
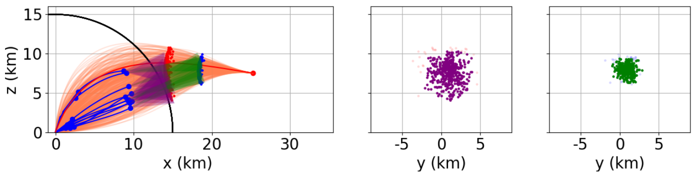


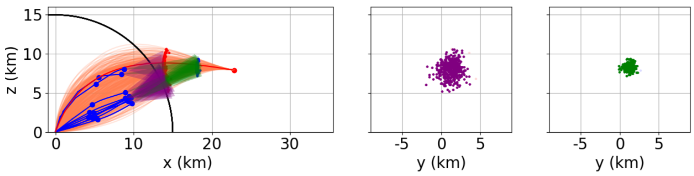
(a) Aim points at initial radar detection ($P_{\text{miss}} = 0.3203$). The first defender will be launched in 0.39 and in the second in 1.41 seconds from initial detection.



(b) Aim points at second radar detection ($P_{\text{miss}} = 0.2056$). The remaining eight defenders will now be launched between 4.47 and 6.19 seconds from initial detection.



(c) Aim points at third radar detection ($P_{\text{miss}} = 0.0734$).



(d) Aim points at fourth radar detection ($P_{\text{miss}} = 0.0102$).

Figure 5. An example with ten defenders. The middle image shows the intercept region (pale red points) and manoeuvre region (purple points) in the yz plane and the rightmost image shows the seek points (pale blue points) and the detection region (green points) in the yz plane.

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