25th International Congress on Modelling and Simulation, Darwin, NT, Australia, 9 to 14 July 2023
mssanz.org.au/modsim2023

Determining the best type of defending missile for cooperative missile defence

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Abstract: Can a team of low-cost defending missiles provide a more cost-effective approach to intercepting a high-performance attacking missile than a single high-cost defending missile? The trajectory of the high-performance attacking missile is unknown and predicting the possible paths brings uncertainties to intercept calculations. Although low-cost defending missiles may have limited abilities to detect and pursue the attack-ing missile, they will have the ability to cooperate with each other to see and intercept the attacking missile.

The objective is to find a cost-effective solution that will maximise the probability of hitting (and stopping) the attacking missile. Two key questions need to be addressed: what is the best strategy for a team of cooperating defending missiles, and what are the cost-effective missile characteristics for a team of cooperating defending missiles?

This paper develops a method for determining which characteristics of defending missiles give the best per-formance. A set of 1728 different defending missile teams were generated by varying the number of missiles, missile speed, seeker performance and missile manoeuvrability. The results show that seeker range has the greatest influence on performance.

Whether a team of low-cost defending missiles is more cost-effective than a single high-performance defending missile depends on the relative costs of the two missile types. Although increasing the number of defending missiles can ensure good performance of a missile defence system, in cases where the unit cost of a low-cost missile remains high, teams of low-cost cooperating defending missiles might not be a cost-effective solution.

Keywords: *Cooperative team defence, air missile defence systems, design of experiments, sensitivity analysis, Pareto front*

1 INTRODUCTION

The aim of this work is to determine whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve performance and flexibility of missile defence systems and be more cost-effective. Figure 1 shows a missile defence system defending against a high-performance attacking missile. The trajectory of the high-performance attacking missile is unknown and predicting the possible paths brings uncertainties to intercept calculations. The defending missiles must prevent the attacking missile from entering the *keep-out* range of the protected target. There are several key events that occur during a missile engagement:

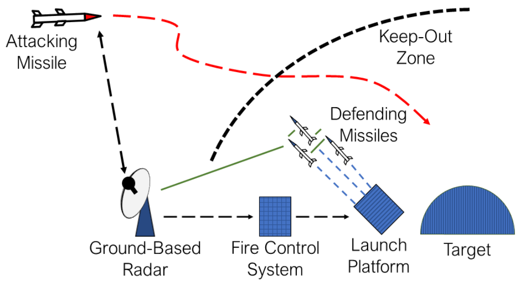


Figure 1. Components of the missile defence system

1. The attacking missile is detected by the ground-based radar. The ground-based radar will provide position and velocity measurements to the fire control system and defending missiles before and after launch.

2. The fire control system will determine how many defending missiles to launch, when to launch them, and where each defending missile should fly to before it turns on its seeker. As the defending missiles fly out towards the attacking missile, measurements from the ground-based radar will be used to refine the possible paths of the attacking missile, which in turn will be used to refine the points that the defending missiles are flying towards.

3. Each defending missile will be equipped with a *seeker* that can detect the attacking missile. The defending missile seekers will provide continuous and more accurate measurements of the attacking missile position and velocity compared to the ground-based radar. The defending missiles will turn on their on-board seekers at the time when the attacking missile is predicted to come within the seeking range of the defending missiles. By this time, the defending missile positions and directions should be organised such that they will be able to see and intercept all possible paths of the attacking missile.

4. Once the attacking missile is detected by at least one defending missile, the team of defending missiles will coordinate to *track* and *intercept* the attacking missile with as many defending missiles as possible.

The objective is to maximise the probability of stopping the attacking missile. The more defending missiles that can see and hit the attacking missile, the higher the probability of stopping the attacking missile.

This paper provides a brief description of the cooperative control strategy (Section 2). Section 3 presents a 'design of experiments' approach [Montgomery, 2017] to generate a number of different defending missile teams and assess their probability of hitting (and stopping) an attacking missile. A simple cost function is formulated to evaluate cost-effectiveness for each defending missile team (Section 3.1). Pareto plots (Section 3.3), an example of a variation of parameters approach (Section 3.4), and main effect screening plots (Section 3.5) are included to help analyse the performance and cost of the different defending missile teams.

2 COOPERATIVE CONTROL STRATEGY

This section provides a brief overview of the cooperative control strategy implemented for a team of defending missiles to maximise the probability of hitting (and stopping) the attacking missile. An early 2D version of the cooperative control strategy was presented at MODSIM2021 [Kapsis et al., 2021]. A 3D implementation of the cooperative control strategy was used to generate the results for this paper [Kapsis et al., 2023].

Consider a simplified 3D problem where curvature of the Earth is ignored. We want to defend against a single attacking missile that is assumed to fly towards a stationary target at position $(0, 0, 0)$. The coordinate system has x pointing east, y pointing north and z pointing up. Suppose the attacking missile is detected at time t_0 by a ground-based radar, and is at position P_0 with velocity $\mathbf{v}_a$. We have up to M defending missiles that can cooperate to seek and intercept the attacking missile. The defending missiles must prevent the attacking missile from entering the *keep-out* range of the protected target. Our method to defend against the attacking missile has the following steps:

1. Given a radar measurement of the position and velocity of the attacking missile, predict possible paths of the attacking missile towards the target. The attacking missile is able to perform aerodynamic manoeuvres and its future path towards the target is unknown to the defending missiles.
2. Define a *seek region* and *intercept region* comprising points on the predicted attacking missile paths where the attacking missile could be at a calculated *seek time* and *intercept time*, respectively.
3. Define an *aim point* for each defending missile to fly towards so the defending missiles will be able to effectively seek and intercept the attacking missile.

Prior to launching the defending missiles, and during their fly-out phase, the ground-based radar will provide periodic updates on the attacking missile's position and velocity. At every *radar update*, the predicted paths of the attacking missile, intercept time, seek time, seek region, intercept region, aim points and defending missile paths will change. The selection of aim points for the defending missiles to fly towards such that the defending missiles are able to maximise their coverage of the seek and intercept regions is a multi-dimensional unconstrained non-linear optimisation problem. The more paths that the defending missile can see and hit, the greater the probability of stopping the attacking missile. The optimisation problem is solved using the Nelder-Mead method [Nelder and Mead, 1965]. Given an engagement scenario and the type and number of defending missiles, the control optimisation determines the best way to operate the defending missiles and the resulting probability of hitting (and stopping) the attacking missile. In the next section we describe how to determine the best type and number of defending missiles for a given engagement scenario.

3 DETERMINING THE BEST TYPE OF DEFENDING MISSILE

We use a 'design of experiments' approach to assess the effectiveness of different types of defending missile teams and determine the best type and number of defending missiles to use, given various defending missile configurations and costs. We are not aware of any literature that addresses this problem. Missile cost information is not available, so to illustrate the method we use a simple cost function that depends on four defending missile performance parameters: speed, maximum lateral acceleration, seeker range and seeker angle.

3.1 Cost function

For each key defending missile parameter we use a general function to map the value of that parameter onto a cost component. The total cost of a missile is the aggregate of these cost components,

$$C = M\left(c_v(v_{\mathrm{m}}) + c_a(a_{\max}) + c_r(r_{\mathrm{s}}) + c_\theta(\theta_{\mathrm{s}})\right).$$

where M is the number of defending missiles, $c_v : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's nominal speed v_{m}, $c_a : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's maximum lateral acceleration $a_{\max}$, $c_r : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's seeker range r_{s}, and $c_\theta : \mathbb{R} \to \mathbb{R}$ is the cost function of the defending missile's seeker angle θ_{s}. The example cost function is based on assumptions that the cost for maximum speed and lateral acceleration is similar to the cost of maximum seeker range and seeker angle, there is a large variation of cost for speed and seeker range, there is a small variation of cost for lateral acceleration, and increasing the seeker angle does not have much impact on cost because small seeker angles require more expensive technology and larger diameter missiles. With guidance from experts and [Fleeman, 2001], we estimated linear cost functions for the four characteristics of the defending missiles:

$$c_v = 0.00052\, v_{\mathrm{m}} + 0.311 \qquad c_a = 0.00340\, a_{\max} + 0.136$$
$$c_r = 0.00002\, r_{\mathrm{s}} + 0.223 \qquad c_\theta = 0.00660\, \theta_{\mathrm{s}} + 0.198.$$

3.2 Experiment design

We used a 'design of experiments' method [Montgomery, 2017] to plan and conduct experiments and analyse the resulting data to understand how cost and probability of hit are influenced by various performance factors of the defending missile team. The factors considered are:

- four defending missile design factors—speed, maximum lateral acceleration, seeker range and seeker angle,
- the keep-out range for the defending missiles,

- the number of defending missiles in the team, and
- the probability of an individual defending missile failing to stop the attacking missile.

It is insightful to include factors with more than two levels. For instance, for lateral acceleration we have included four levels ($1g, 2g, 5g$ and $10g$). The optimisation method used in Section 2 to determine the probability of hit for a team of defending missiles is fast enough that we could do a full factorial multi-level design for this analysis, whereby all possible combinations of the levels of the seven factors presented in Table 1 were investigated. The computation time to evaluate all 1728 experiments was 8.66 hours on a 2.9 GHz 6-Core Intel Core i9 processor.

Table 1. Factors and levels for missile type analysis.

Factor	Levels
keep-out range (km)	15, 30
number of missiles	$1, 2, 5, 10$
probability of fail	$0.0, 0.4, 0.9$
speed ($\mathrm{km\,s^{-1}}$)	$0.2, 0.6, 1$
lateral acceleration ($\mathrm{m\,s^{-2}}$)	$1g, 2g, 5g, 10g$
seeker range (km)	5, 10, 20
seeker angle (degrees)	$10, 20$

The engagement scenario has one attacking missile and a team of defending missiles. The attacking missile travels at a constant speed of $0.6\,\mathrm{km\,s^{-1}}$ and starts from an altitude of 7 km and at a horizontal distance that depends on the keep-out range and the speed of the defending missiles, so that there is always sufficient time to fly out and intercept the attacking missile. The following results evaluate the optimised probability of hit calculated at the time the defending missiles are launched. Typically, the probability of hit will improve after the defending missiles are launched.

3.3 Pareto front

We are interested in defending missile configurations that minimise cost and maximise the probability of hitting (and stopping) the attacking missile. A defending missile configuration is said to *Pareto-dominate* another configuration if the cost is at least as low, the performance is at least as high, and one or both of cost or performance is strictly better. The set of all non-dominated, or *Pareto-efficient*, solutions form a *Pareto front*. For every solution that is not on the Pareto front, there is a better solution on the front.

Figure 2 shows the cost and probability of hit for every defending missile team investigated. The three columns correspond to different probabilities of fail for individual missiles. Each subplot shows the relationship between cost and probability of hit for each team of defending missiles. The points connected by the purple line in each subplot form the Pareto front. The pale points in the interior of the region are the Pareto-dominated solutions. Adding more defending missiles increases the probability of hit. When the probability of fail is 0.0, a single defending missile is able to provide the best performance at the lowest cost. Some trends that can be observed from the graphs are:

- Keep-out range does not have much of an influence on the results for the values considered.
- When each defending missile has a low probability of fail then teams of 1 or 2 missiles achieve a high probability of hit. As the probability of fail of individual missiles becomes higher, more missiles are required to achieve an acceptably high probability of hit.
- The cost of the defending missile teams fall into distinct bands determined by the number of missiles in the team. On the Pareto front, increasing the number of missiles in the team always increases performance but also always increases the cost.
- At the top edge of the Pareto fronts, increasing the number of missiles can give a large increase in cost for a small improvement in performance.

- At the left edge of the Pareto fronts, particularly when the probability of failure is low, a small increase in cost can give a large increase in performance.

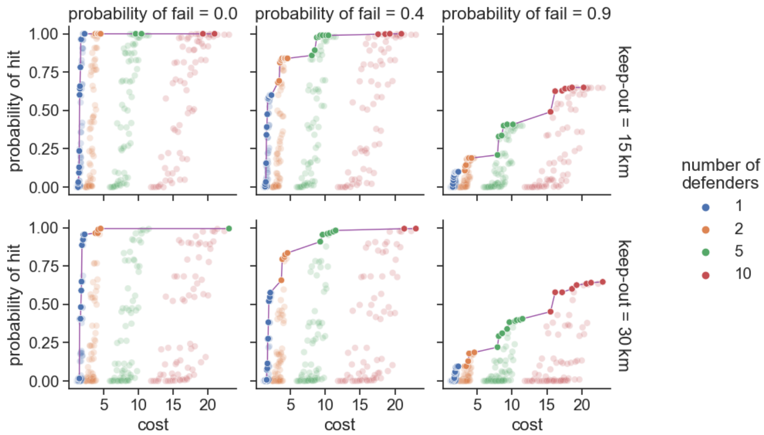


Figure 2. Cost and probability of hit for the defending missile teams. The points connected by the purple line form the Pareto front. Each subplot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

3.4 Sensitivity analysis on a Pareto-efficient solution

This section gives a sensitivity analysis on a Pareto-efficient solution to investigate which missile factors are important. The base solution is a Pareto-efficient solution for the case when keep-out range is 15 km and probability of fail is 0.4. This solution is shown in Figure 3, and has a probability of hit of 0.3924 and cost 1.6359. The defending missile team has one missile with a speed of 0.6 km s^{-1}, maximum lateral acceleration of $5g$, seeker range of 10 km and seeker angle of 10 degrees.

We performed a sensitivity analysis by varying each parameter, one at a time, using the results from the 1728 experiments, and evaluating the change in performance or cost per change in factor level (shown in Table 1). The results are shown in Table 2. Increasing the seeker angle to 20 degrees has the greatest performance improvement per increase in cost and gives another Pareto-efficient solution. Increasing the seeker range to 20 km also gives another Pareto-efficient solution, and the most gain in probability of hit. Increasing the speed to 1 km s^{-1} does not give a Pareto-efficient solution. Increasing the number of missiles to two is the most costly variation, and does not give a Pareto-efficient solution. Increasing the maximum lateral acceleration to $10g$ increased cost without giving a performance benefit.

3.5 Screening experimental factors

Screening is a technique for identifying factors that have significant effects on the objective function [Montgomery, 2017]. Figure 4 show the variation of probability of hit for speed, maximum lateral acceleration,

Table 2. Sensitivity analysis about a Pareto-efficient solution with one missile, constant speed of 0.6 km s^{-1}, maximum lateral acceleration of $5g$, seeker range of 10 km and seeker angle of 10 degrees.

Factor F	$\Delta P_{\text{hit}}/\Delta F$		$\Delta\text{Cost}/\Delta F$		$\Delta P_{\text{hit}}/\Delta\text{Cost}$
number of missiles	0.2086	per missile	1.6359	per missile	0.1275
speed	0.2160	per km s^{-1}	0.5188	per km s^{-1}	0.4164
lateral acceleration	-0.0002	per 9.81 m s^{-2}	0.0336	per 9.81 m s^{-2}	-0.0072
seeker range	0.0186	per km	0.0223	per km	0.8328
seeker angle	0.0083	per degree	0.0066	per degree	1.2545

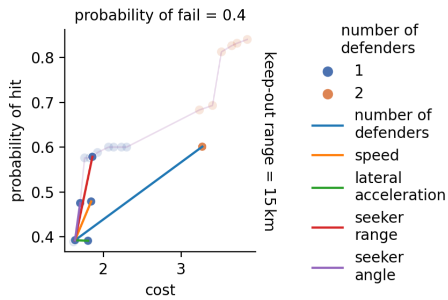


Figure 3. A section of the Pareto front for keep-out range 15 km and probability of fail 0.4. The points along the Pareto front are connected by a pale purple line. Given the Pareto-efficient solution with probability of hit 0.3924 and cost 1.6359 units, we have identified the solutions where: the number of missiles is increased (blue line), speed is increased (orange), maximum lateral acceleration is increased (green line), seeker range is increased (red line) and seeker angle is increased (purple line).

seeker range and seeker angle. The lines in each plot show the mean probability of hit at each level of each factor, where each color indicates the number of missiles. The mean probability of hit for a factor and level considers performance over all levels of all other factors. By observing these plots we can determine the influence of each factor on probability of hit. A steep line indicates that the factor has significant influence in probability of hit.

Figure 4a shows the interaction of mean probability of hit with defending missile speeds of 0.2, 0.6, and $1\,\mathrm{km\,s^{-1}}$. The attacking missile is traveling at a constant speed of $0.6\,\mathrm{km\,s^{-1}}$. When keep-out range is 15 km, increasing the defending missile speed from $0.2\,\mathrm{km\,s^{-1}}$ to $0.6\,\mathrm{km\,s^{-1}}$ has a significant influence on probability of hit. Further increasing the defending missile speed to $1\,\mathrm{km\,s^{-1}}$ gives a smaller increase in performance.

Figure 4b shows the interaction of mean probability of hit with defending missile maximum lateral acceleration levels of $1g$, $2g$, $5g$, and $10g$. The attacking missile is capable of maximum lateral acceleration of $5g$ manoeuvres. Performance increases with maximum lateral acceleration, but the improvement reduces as maximum lateral acceleration increases.

Figure 4c shows the interaction of mean probability of hit with defending missile seeker ranges of 5, 10, and 20 km. These plots show a significant increase in performance with seeker range across the entire range of values.

Figure 4d shows the interaction of mean probability of hit with defending missile seeker angles of 10 and 20 degrees. These plots show that seeker angle has relatively little influence on probability of hit.

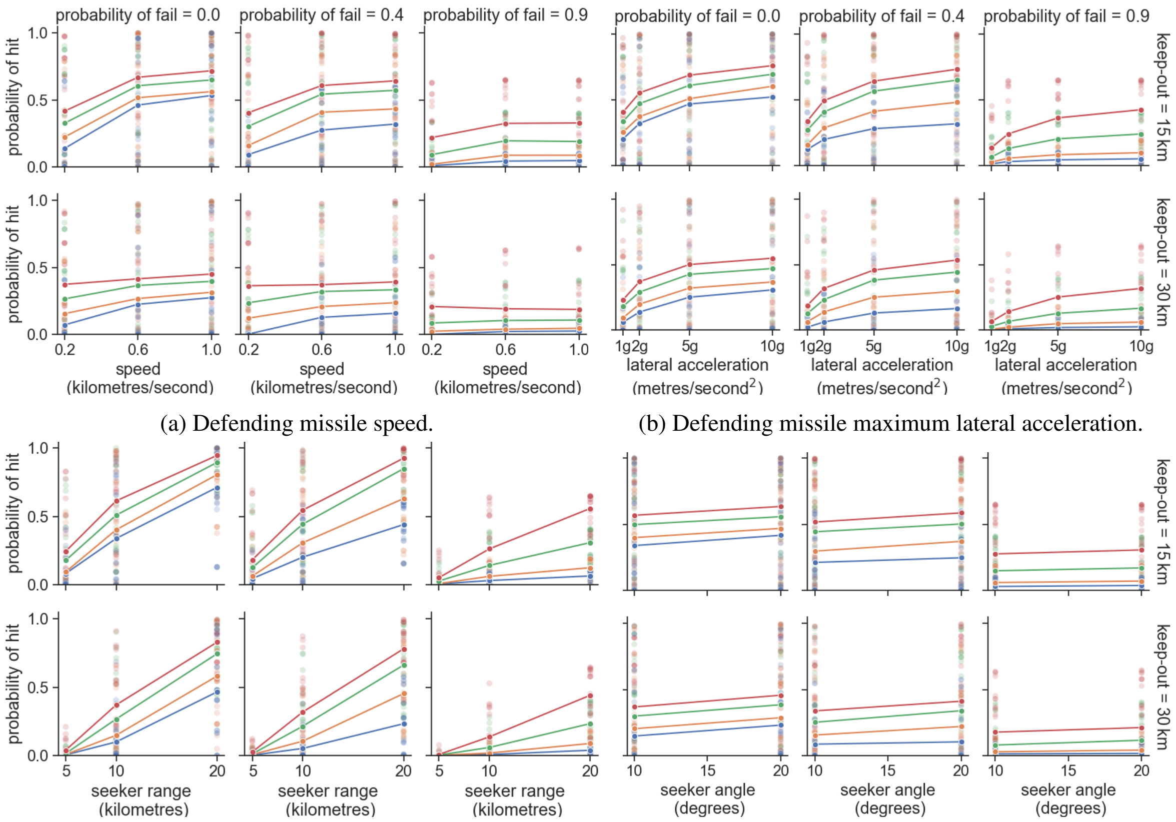
The variation of cost for each factor is not considered in this analysis. However, for a missile team to have good performance it would be reasonable to have a high seeker range, and speed and lateral acceleration at least equivalent to the attacking missile.

4 Conclusions and Future Work

A 'design of experiments' approach can be used to assess the performance and cost of different configurations of defending missiles, to determine whether replacing a single high-cost defending missile with a team of cooperating low-cost defending missiles can improve performance of missile defence systems and be more cost-effective.

A set of 1728 experiments was used to evaluate the influence of keep-out range, number of defending missiles, probability of fail, speed, maximum lateral acceleration, seeker range and seeker angle on the probability of hit at time of launch. In our engagement scenario example, seeker range had the most influence on the probability of hitting (and stopping) the attacking missile, and seeker angle the least. Increasing the number of defending missiles had the strongest influence on cost. Having more than one missile provides a performance benefit when the probability of failure for an individual missile is high. However, for the cost functions we used, there were many situations where it is better to have a small number of defending missiles with higher performance rather than a larger number of defending missiles with lower performance.

The effectiveness of our cooperative control strategy could be enhanced by incorporating higher fidelity mod-



(a) Defending missile speed.

(b) Defending missile maximum lateral acceleration.

(c) Defending missile seeker range.

(d) Defending missile seeker angle.

Figure 4. Interactions of mean probability of hit with defending missile parameters. Each plot shows results for probability of fail equal to 0 (left), 0.4 (middle) and 0.9 (right), and when keep-out range is 15 km (top) and 30 km (bottom). Each color shows results for 1 (blue), 2 (orange), 5 (green) and 10 (red) missiles in the defending missile team.

els of the missile defence system, incorporating radar errors, and incorporating terminal guidance for the defending missiles. Doing the same analysis with more realistic performance parameters and costs may give a different result. Defending against multiple attacking missiles with multiple cooperating defending missiles may be another application to investigate.

Acknowledgments

This project was conducted as part of a Master of Research program at the University of South Australia and was supported by the Australian Government Research Training Program (RTP) Scholarship and Defence STEM Cadetship Program. We thank the team at DSTG for modelling advice.

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