

Using logic-based Benders' decomposition approach for a two-stage integer stochastic staffing and scheduling problem

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Abstract: Our study presents an integrated staffing and scheduling model, which is a two-stage stochastic problem with demand as the random variable. In the first stage, we focus on hiring and training permanent staff. In the second stage, we schedule the existing permanent workforce and hire temporary staff to meet the actual demand. The first stage involves binary variables, and the second stage encompasses both binary and integer variables.

To solve the deterministic equivalent form of a linear two-stage stochastic optimization problem, Benders' decomposition is a commonly used solution technique. This approach divides the original problem into a separated main problem and subproblem and generates Benders' cuts from Lagrange multipliers of the subproblem. However, in combinatorial optimization problems like staffing and scheduling problems, Lagrange multipliers are not readily available.

The most efficient technique for solving stochastic combinatorial optimization problems is using the logic-based Benders' decomposition technique, which generates the cuts from structural analysis of the subproblem. However, the main challenge in applying this approach is the lack of connection between the main and subproblems. To address this issue, we introduce two main series of constraints. The initial set of constraints elucidates the trade-off observed between the augmentation of permanent staff skills and the reduction in costs associated with temporary workforce expenditure. The subsequent set of constraints signifies the interplay between the acquisition of specialized skills by permanent staff members and the resulting savings in temporary workforce expenditure.

The magnitude of the solution space for the master problem has a noteworthy role in the efficiency of a Benders' decomposition algorithm. Consequently, we have endeavoured to linearise our Benders' cuts in order to diminish the maximum feasible extent within the solution space of the master problem. Additionally, to further diminish the solution space of the master problem, an additional sequence of Benders' cuts can be formulated to preclude the model from training the workforce in undesirable locations. Our experimental results demonstrate that the proposed method can outperform Gurobi by several degrees of magnitude, which computes direct solutions of the monolithic deterministic equivalent model. The results of this investigation can be generalised to be applicable to resource and flexible resource scheduling.

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