# Modelling of the Activated Sludge process with a stratified settling unit

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**Abstract:** The activated sludge process is the most widely used process for the biological treatment of domestic and industrial wastewaters. Wastewater treatment plants using the activated sludge process are widely used in developed and developing countries. The activated sludge model number 1 (ASM #1) is an internationally accepted standard for activated sludge modelling. It describes nitrogen and chemical oxygen demand within suspended-growth treatment processes, including mechanisms for nitrification and denitrification.

We analysed the biological treatment of wastewater when a single aerated reactor was investigated. The residence time was used as the bifurcation parameter and investigated how the autotrophic and heterotrophic biomass concentrations varied.

The configuration analysed consisted of a continuously stirred tank reactor (CSTR) with an attached settling tank. The settling tank concentrated the particulate matter that flowed from the reactor, before being returned back into the reactor. We considered two different models of the settling tank and compared the behaviour. The first was a simple settling tank, where it was assumed that the particulate matter was concentrated by a calculated factor instantaneously. The second was a stratified settling tank, where the tank was divided into a number of layers. The settling occurred through gravity settling and this lead to a separating flow, where the feed is clarified towards the top of the tank and thickened towards the bottom.

The result was that both models lead to three types of behaviour

- 1. For low residence times, the flow rate was too fast for the heterotrophic or autotrophic biomass to develop and therefore there was no reaction occurring in the reactor. We call this the Washout state, as the wastewater flows through the system without any change.
- 2. For slightly higher residence times, a transcritical bifurcation occurs where the heterotrophic biomass concentration first becomes non-zero.
- 3. Finally, there is a higher residence time when a second transcritical bifurcation occurs where both the heterotrophic and autotrophic biomass concentrations become non-zero.

We found that for the stratified settling tank, both transcritical bifurcations occurred earlier than for the simple settling tank (0.029 days compared to 0.053 days for the first bifurcation and 0.117 days compared to 0.389 days for the second bifurcation). When the configuration with the stratified settling tank was in the Washout state, the settling tank was acting as a dilution tank, with a concentration factor below one.

Keywords: Activated sludge, modelling, recycling, stirred tank, wastewater, settling tank

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# **1 INTRODUCTION**

The activated sludge process is the most commonly used aerobic process for the biological treatment of both domestic and industrial wastewaters (Wei et al. 2003). As such, most wastewater treatment plants contain a unit employing the activated sludge process. The key to this process is the presence of highly concentrated micro-organisms, typically 2–4 g/L, present in the form of flocs, which grow though consuming organic pollutants. The mixture of flocs and particulate matter is known, for historical reasons, as "activated sludge". Without sufficient quantities of sludge, i.e. micro-organisms, the process cannot work.

The simplest reactor configuration for the activated sludge process entails the use of two steps: a series of biological reactors and a settling tank (or clarifier). In the former the pollutants are degraded by microorganisms (the active agent that puts the 'activated' into 'activated sludge'). Inside each of the bioreactors the micro-organisms flocculate to form settleable solids. These solids are removed from the effluent stream by sedimentation in a settling tank and then returned to the first reactor in a more concentrated culture. The use of a clarifier ensures the presence of a culture of highly concentrated micro-organisms and is one of the keys to the success of the activated sludge process.

Modelling has become an important tool to develop an understanding of the processes that govern the behaviour of the activated sludge process. A widely used model that describes the biological processes occurring in the activated sludge process is the activated sludge model #1 (ASM #1) (Henze et al. 1987). This consists of a biochemical model for the particulates and soluble materials and a sub-model describing nitrifying processes.

We investigate the steady-state behaviour of the ASM #1 as a function of the residence time. The residence time is defined as the volume of the reactor divided by the influent flow rate. It represents the average amount of time that a soluble compound remains in the reactor. From the steady-state values of the state variables we can calculate the corresponding steady-state values of various quantities associated with the operation of the activated sludge process, such as the chemical oxygen demand, the total suspended solids, total nitrogen and total inorganic nitrogen. We will measure the performance of the system using the total nitrogen. The total nitrogen should be reduced to below a specific level, as excess amount of nitrogen in a waterway can lead to low levels of dissolved oxygen.

# 2 MATERIALS AND METHODS

We implement the Activated Sludge Model number 1 (ASM #1) (Henze et al. 1987) in a configuration with a single reactor and a settling tank (part of the exit stream from the settling tank is discharged as waste). A schematic of the reactor configuration is shown in Figure 1. See Henze et al. (1987) for a detailed description



Figure 1. A schematic of the bioreactor configuration with a settling tank.

of the ASM #1 and to Nelson et al. (2019) for a full nomenclature and a complete listing of parameter values. The latter also provides the differential equations for a single reactor connected to an ideal settling tank. The ideal settling tank is characterised by three parameters. These are the recycle ratio (R), the concentration factor of particulates (C), and the fraction of the non-effluent stream leaving the settling tank that is discharged to waste (w).

The ASM #1 incorporates differential equations for six soluble and six particulate components. The six soluble components are: inert soluble organic material, readily biodegradable soluble substrate, soluble oxygen, soluble nitrate & nitrite nitrogen (one component), soluble ammonium ( $NH_4^+$  and NH) nitrogen, and soluble biodegradable organic nitrogen. The particulate components are: particulate inert organic matter, slowly biodegradable particulate substrate, active heterotrophic particulate biomass, active autotrophic particulate biomass, non-biodegradable particulate products arising from biomass decay, and particulate biodegradable organic nitrogen.

## 2.1 Formulation of reaction rates

The ASM #1 includes eight fundamental processes that describe nitrogen and chemical oxygen demand within suspended-growth treatment processes, These are: (1) aerobic growth of heterotrophic biomass, (2) anoxic growth of heterotrophic biomass, (3) aerobic growth of autotrophic biomass, (4) decay of heterotrophic biomass, (5) decay of autotrophic biomass, (6) ammonification of soluble organic nitrogen, (7) hydrolysis of entrapped particulate organic matter and (8) hydrolysis of entrapped organic nitrogen. These processes includes mechanisms for nitrification and denitrification

In writing the equations we use the symbol  $S_O$  to denote the concentration of soluble oxygen in the reactor and the symbol  $S_j$  (j = 1...5) to denote the concentration of the five other soluble components in the reactor. We denote the concentration of the six particulate components in the reactor by  $X_l$  (l = 1...6). The rate functions are specified in a column vector. The *i*th row of this vector is written  $\mathbf{f}_i(\mathbf{S}, \mathbf{X})$ . The *i*th reaction rate depends upon the corresponding concentration of the soluble and particulate components. In this formulation the variable  $\mathbf{S}$  includes all six of the soluble components. The reaction rate expression may contain as few as one of the twelve state variables. Two of the state variables appear in no reaction rate expressions.

For each of the state variables there is a stoichiometric column vector  $\alpha_{i,j}$ . The second index (j = 1, 2, ..., 12) identifies the state variables being considered. The first index (i = 1, 2, ..., 8) indicates the process being considered (as numbered above). If the entry  $\alpha_{i,j}$  is positive, then component j is a product in reaction i. If the entry  $\alpha_{i,j}$  is negative, then component j is a reactant in reaction i. If the entry  $\alpha_{i,j}$  is zero, then component j is neither a product nor a reactant in reaction i.

# 2.2 Characterisation of the activated sludge process

The concentrations of each of the twelve state variables are determined in each reactor. Various linear combinations of the state variables are of particular interest. These combinations define different process variables, such as chemical oxygen demand (COD), total suspended solids (TSS), total nitrogen (TN), and total inorganic nitrogen. These variables may be evaluated in the influent stream, in each of the reactors, in the sludge waste stream, and in the effluent stream. We consider the total nitrogen concentration in the effluent stream.

# **3 MODEL EQUATIONS**

#### 3.1 Model equations in the first reactor with a simple settling tank

Rate of change of soluble components (excluding oxygen) in the reactor (j = 1...5)

$$\frac{\mathrm{d}S_j}{\mathrm{d}t} = \frac{1}{\tau} \left( S_{j,\mathrm{in}} - S_j \right) + \frac{R}{\tau} \left( S_j - S_j \right) + \sum_{i=1}^8 \alpha_{i,j} \cdot \mathbf{f}_i \left( \mathbf{S}, \mathbf{X} \right).$$
(1)

Rate of change of soluble oxygen in the reactor

$$\frac{\mathrm{d}S_{\mathrm{O}}}{\mathrm{d}t} = \frac{1}{\tau} \left( S_{O,\mathrm{in}} - S_{\mathrm{O}} \right) + \frac{R}{\tau} \left( S_{\mathrm{O}} - S_{\mathrm{O}} \right) + K_{\mathrm{L,a}} \left( S_{\mathrm{O,max}} - S_{\mathrm{O}} \right) + \sum_{i=1}^{8} \alpha_{i,O} \cdot \mathbf{f}_{i} \left( \mathbf{S}, \mathbf{X} \right).$$
(2)

Rate of change of particulate components in the reactor  $(l = 1 \dots 6)$ 

$$\frac{\mathrm{d}X_l}{\mathrm{d}t} = \frac{1}{\tau} \left( X_{l,\mathrm{in}} - X_l \right) + \frac{R}{\tau} \left( \mathcal{C}X_l - X_l \right) + \sum_{i=1}^8 \alpha_{i,l} \cdot \mathbf{f}_i \left( \mathbf{S}, \mathbf{X} \right).$$
(3)

Note that there are zero terms in equations (1) and (2), where the soluble components feed straight through the settling tank, back into the reactor. In these equations the parameters are:  $K_{L,a}$ , the oxygen transfer coefficient;

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Figure 2. Schematic of the stratified settler.

R, the recycle ratio of the settling tank;  $S_{j,in}$ , the concentration of soluble component j in the influent;  $S_{O,in}$ , the concentration of soluble oxygen in the influent;  $S_{O,max}$ , the maximum concentration of soluble oxygen;  $X_{l,in}$  is the concentration of the particulate components l in the influent, and  $\tau$ , the residence time in the reactor. The influent concentrations are assumed to be fixed. The concentration factor for particulates in the settling tank, C, is defined as

$$\mathcal{C} = \frac{R+1}{R+w},\tag{4}$$

where w is the wasted fraction of sludge.

## 3.2 Model equations in the first reactor with a stratified settling tank

We extend the simple settling tank to a stratified settling tank, as described in Flores-Tlacuahuac et al. (2009). The idea is that rather than the particulate components being instantaneously concentrated as they pass through the settler, the flow from the reactor is fed into the settler, where gravity settling occurs with the particulates falling towards the bottom layer of the settler and a clarified feed taken from the top layer of the settler. A schematic is shown in Figure 2. The model is similar inside the reactor, with the feed from the bottom layer P

getting fed back into the reactor, changing the recycle terms in (1), (2) and (3) from 
$$\frac{R}{\tau}(S_j - S_j)$$
,  $\frac{R}{\tau}(S_O - S_O)$  and  $\frac{R}{\tau}(CX_l - X_l)$  to  $\frac{R}{\tau}(S_{j,st} - S_j)$ ,  $\frac{R}{\tau}(S_{O,st} - S_O)$  and  $\frac{R}{\tau}(X_{l,st} - X_l)$ , respectively.

Rate of change of soluble components (excluding oxygen) in the reactor (j = 1...5)

$$\frac{\mathrm{d}S_j}{\mathrm{d}t} = \frac{1}{\tau} \left( S_{j,\mathrm{in}} - S_j \right) + \frac{R}{\tau} \left( S_{j,st} - S_j \right) + \sum_{i=1}^8 \alpha_{i,j} \cdot \mathbf{f}_i \left( \mathbf{S}, \mathbf{X} \right).$$
(5)

Rate of change of soluble oxygen in the reactor

$$\frac{\mathrm{d}S_{\mathrm{O}}}{\mathrm{d}t} = \frac{1}{\tau} \left( S_{O,\mathrm{in}} - S_{\mathrm{O}} \right) + \frac{R}{\tau} \left( S_{\mathrm{O},st} - S_{\mathrm{O}} \right) + K_{\mathrm{L,a}} \left( S_{\mathrm{O,max}} - S_{\mathrm{O}} \right) + \sum_{i=1}^{8} \alpha_{i,O} \cdot \mathbf{f}_{i} \left( \mathbf{S}, \mathbf{X} \right).$$
(6)

Rate of change of particulate components in the first reactor  $(l = 1 \dots 6)$ 

$$\frac{\mathrm{d}X_l}{\mathrm{d}t} = \frac{1}{\tau} \left( X_{l,\mathrm{in}} - X_l \right) + \frac{R}{\tau} \left( X_{l,st} - X_l \right) + \sum_{i=1}^8 \alpha_{i,l} \cdot \mathbf{f}_i \left( \mathbf{S}, \mathbf{X} \right), \tag{7}$$

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where the subscript st denotes the corresponding component flowing out of the settling tank. In the settling tank, there are  $N_s$  stratification layers and the feed from the reactor flows into the settling tank at layer  $N_f$ . In all but the bottom layer, there is gravity settling from the layer above. In addition, there is a separating flow above and below the feed layer. These two processes lead to clarification at the top of the settling tank and thickening at the bottom. The flow from the bottom layer then feeds back into the reaction. The model equations inside the settling tank are given below.

Top level  $j = N_s$ 

$$\frac{dX_{N_s}}{dt} = \frac{v_{up}(X_{N_s-1} - X_{N_s}) + F_{N_s-1}}{h}.$$
(8)

Clarification levels  $N_f < j < N_s$ 

$$\frac{dX_j}{dt} = \frac{v_{up}(X_{j-1} - X_j) + F_{j+1} - F_j}{h}.$$
(9)

Feed level  $j = N_f$ 

$$\frac{dX_{N_f}}{dt} = \frac{(v_{up} + v_{down})(X_{feed} - X_{N_f}) + F_{N_f + 1} - F_{N_s}}{h}.$$
(10)

Thickening levels  $1 < j < N_f$ 

$$\frac{dX_j}{dt} = \frac{v_{down}(X_{j+1} - X_j) + F_{j+1} - F_j}{h}.$$
(11)

Bottom level j = 1

$$\frac{dX_1}{dt} = \frac{v_{down}(X_2 - X_1) + F_2}{h},$$
(12)

where  $v_{up}$  and  $v_{down}$  are the flow rate up and down,  $F_j$  is the flow from level j due to gravity settling, h is the height of each level. See Flores-Tlacuahuac et al. (2009) for the full nomenclature and a complete listing of parameter values.

# 4 RESULTS AND DISCUSSION

The systems (1)–(3) and (5)–(12) are integrated numerically using MATLAB until a steady state, or equilibrium, is reached. From a starting concentration of each of the state variables, the state variable will evolve to one of a number of solution paths. In general, there will be only one possible solution path for a given set of parameters. As the system parameters change, the solution path followed can change if the solution path becomes unstable when one of the eigenvalues of the linearised system becomes positive. For this paper, the only parameter which is being varied is the residence time ( $\tau$ ). All other parameters are being kept constant.

We compared the behaviour of the two settling tank models by considering the variation of the autotrophic biomass  $X_{B,A}$ , heterotrophic  $X_{B,H}$  biomass concentrations in the reactor with respect to the residence time  $\tau$ . From Figure 3, there are similar features, in that for low residence times (corresponding to fast flow rates), the biomass concentrations are zero and the wastewater is not broken down at all. We call this the Washout condition. As the residence time increases, the heterotrophic biomass concentration is non-zero. At even higher residence times, the autotrophic biomass concentrations first become non-zero and another solution path is followed. For the simple settling tank, the bifurcation points are at 0.029 and 0.117 days, respectively. Another difference is that the maximum heterotrophic biomass concentration increases significantly between the two.

The concentration factor of the particulate components for the two settling tanks are shown in Figure 4. For the simple settler tank, the concentration factor is given in (4). For the stratified settler tank, the concentration factor is the ratio of the particulate concentrations flowing into and flowing out of the settler tank (in layers  $N_f$  and 1 in Figure 2, respectively). For lower residence times, when the Washout condition occurs in the configuration with the stratified settling tank, the concentration factor is less than one, which would be diluting rather than concentrating the particulate components. The concentration factor decreases as a function of the residence time.



**Figure 3**. Variation of autotrophic and heterotrophic biomass concentrations (mg COD/l) in a reactor with the simple settling tank (left) and the stratified settling tank (right).



**Figure 4**. Comparison of the concentration factor (C) of the simple settler tank (dashed line) and the stratified settling tank (solid line).

# **5 FUTURE WORK**

There does appear to be some improvements to be made by including a stratified settling tank in the ASM#1 model, as compared to the simple settling tank as discussed in Nelson et al. (2019). However, further work would be needed to calibrate the two models, as the are some variations between the parameter values for the wastewater in Nelson et al. (2019) and Flores-Tlacuahuac et al. (2009).

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