# Impact of search space discretisation on water distribution system design optimisation

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**Abstract:** Water distribution systems (WDSs) are essential components of both agricultural and urban infrastructure systems. In recent years, there have been an increase in energy consumption and associated costs and greenhouse gas (GHG) emissions for water supply and distribution. This has led water utilities to optimise their systems not only for economic benefits but also to reduce environmental impact. Real-world WDSs are topologically and dimensionally complex systems with many interconnected components including pipes, pumps and storages. These systems are often associated with a large search space in the optimisation process. Therefore, the question arises as to whether the search space can be reduced, and yet effective optimisation still be achieved.

In this study, a hydraulic-power-based search space reduction method (power-based SSR method) has been used to reduce the optimisation search space by grouping pipes with similar hydraulic power capacity. The impact of search space discretisation on the optimisation performance has been investigated using a real-world WDS with 432 pipes. A multi-objective optimisation (MOO) problem has been formulated. The objectives considered include the minimisation of both the total life cycle cost and total life cycle greenhouse (GHG) emissions over the system design life. Pipe diameters are the decision variables. Various problem formulations with decision variable numbers ranging from 5 to 432 have been compared against two performance indicators: (1) the number of evaluations needed to achieve convergence, where large values indicate lower optimisation efficiency; and (2) the Hypervolume Indicator (HI), where larger HI values indicate better optimisation convergence.

Results show that first, trade-offs between the two objective function values with clear Pareto fronts have been observed. With the increase in the number of decision variables, better convergence and smaller minimum

objective function values can be achieved. Second, for the two performance indicators, an increase in the number of decision variables in general leads to an increase in both the number of evaluations needed for convergence (i.e., reduced optimisation efficiency) and the Hypervolume Indicator (HI) value of the final optimal solutions (i.e., improved convergence). In addition, as shown in Figure 1, there are also trade-offs observed between the speed of convergence and associated performance. Better convergence requires more optimisation effort, with an increased degree of search space discretisation.



Figure 1. Relationship between two performance indicators

# *Keywords:* Water distribution systems, pipe sizing, hydraulic power, search space reduction; multi-objective optimisation

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# 1. INTRODUCTION

Water distribution systems (WDSs) are essential components of both agricultural and urban infrastructure systems. The rise in water demand due to population growth has resulted in increased energy consumption, associated costs and greenhouse gas (GHG) emissions worldwide in recent years. Therefore, water utilities are exploring ways to optimise these systems not only to promote economic benefits but also to reduce environmental impact.

WDSs are topologically and dimensionally complex systems that include various interconnected components (Abdul Gaffoor, 2017). The optimal design of WDSs commonly involves selecting the best combination of system components such as pipes, pumps and storages, along with their appropriate sizes and locations to attain the minimum total life cycle cost (Bagloee et al., 2018). In addition, some studies have adopted multi-objective optimisation methods to simultaneously minimise GHG emissions (Stokes et al., 2014; Wu et al., 2012).

Optimisation tools are frequently employed to assist the design of WDSs due to their complexity and substantial life cycle costs (Wu et al., 2010; Wu et al., 2008). Deterministic (also known as classic) and metaheuristic algorithms are two common types of optimisation techniques used for WDS design optimisation. Compared to deterministic optimisation methods, metaheuristic algorithms such as Evolutionary Algorithms (EAs), have demonstrated superior performance in resolving complex problems with more decision variables and constraints (Maier et al., 2019). Although metaheuristics are typically associated with a greater computational cost, they have an increased chance of discovering optimal or near-optimal solutions due to their exploratory nature (Coelho and Andrade-Campos, 2012). However, their applications to complicated WDSs still remain a challenging task as the search space in these real-world problems is often large (Wolpert and Macready, 1997).

Traditionally, it has been considered by some water utilities that pipes that deliver the same flow capacity may have the same diameter in the design process. However, with the consideration of pipe elevations and the total head losses from the pump station, these pipes under similar flow conditions may end up with different optimal solutions. For instance, under the same desired pressure requirement and the design flow, pipes that are located at lower elevations and closer to the pump station may need smaller diameters in comparison with those located higher and further. In this paper, a hydraulic-power-based search space reduction method (power-based SSR method) (Zhao et al., 2023) has been applied to group pipes with similar hydraulic power capacity based on both the head and flow of a pipe (Park et al., 1998; Vaabel et al., 2006; Wu et al., 2011). This involves not only the maximum flow through the pipe but the residual pressure head at the outlet of the pipe, which is an indicator of the relationship between the ground elevation, upstream head losses and a certain pumping head provided by the pump station. This method is proposed to reduce the search space size in real-world optimisation problems. However, to what extent the search space should be reduced or discretised, and what can be the ideal number of decision variables considering realistic computational resources, remain an open question for decision-makers when using this method.

In this paper, a multi-objective optimisation problem has been formulated to investigate optimal economic and environmental solutions for different numbers of decision variables using the power-based SSR method. Optimisation performances have also been evaluated against different numbers of decision variables formulated using the developed method.

# 2. CASE STUDY SYSTEM

The system used in this study is a pressurised irrigation network located in the Robinvale irrigation district in north-western Victoria, Australia. The location of the study area and the network are shown in Figure 2. Raw water is pumped from the Murray River via a high-pressure pump station on the southern bank of the river and then directly distributed through the pipeline system to customers for irrigation and domestic use. There is no water storage in the system. This region primarily grows table grapes, which require substantial amounts of water for irrigation (Lower Murray Water, 2019). The network comprises 433 pipes and 244 irrigation outlets. The minimum pressure head required for irrigation water delivery at user outlets is 35 m. In this study, pipe diameters are optimised. Relevant data was provided by the local water authority Lower Murray Water (LMW). An EPANET model (Rossman, 2000) serves as the simulation model for the system.



Figure 2. Location of the study area and the Robinvale high-pressure system in Victoria, Australia

# 3. METHODS

# 3.1. Problem formulation

This study presents a multi-objective optimisation (MOO) problem to minimise both the total life cycle cost and total life cycle GHG emissions of the case study system. The decision variables of the optimisation problem are pipe diameters. The system incorporates 5 MW behind-the-meter (BTM) solar PV to reduce energy consumption from the centralised energy supply grid and associated GHG emissions. The NSGA-II algorithm (Deb et al., 2002) has been considered in this problem and was implemented in a Python-based optimisation package called 'pymoo' (Blank and Deb, 2020). In addition, a Python wrapper (Open Water Analytics, 2020) has been installed to call functions in EPANET Programmer's Toolkit (Rossman, 1999).

The first objective function (OF1) is to minimise the total life cycle cost, which is given by

$$min \, OF1: \, LCC \,= \, CC + OC + PRC + SRC \tag{1}$$

where LCC = total life cycle cost, CC = capital costs, OC = operating costs, and PRC = pump refurbishment costs and SRC = solar panel replacement costs, respectively. It is assumed that the system has the same lifespan as pipes, which is 100 years (Water Services Association of Australia, 2011). Capital costs primarily involve purchasing and constructing pipes, pumps and solar panels. Operating costs mainly include expenses on electricity purchased from the grid when the energy required exceeds solar energy production. Pumps and solar panels are assumed to be refurbished and replaced every 20 years (Water Services Association of Australia, 2011) and 25 years (Prasad et al., 2005), respectively. Present value analysis (PVA) has been conducted for operating costs, as well as pump and solar PV replacement costs, with a social discount rate of 1.4% used (Stern et al., 2006).

The second objective function (OF2) is to minimise the total life cycle GHG emissions, which is given as

$$min \, OF2: \, LCGHG \,= CGHG \,+ \, OGHG \qquad (2)$$

where LCGHG = total life cycle, CGHG = capital GHG emissions and OGHG = operating GHG emissions, respectively. Capital GHG emissions mainly arise from the acquisition of raw materials and the production process of pipes and solar panels. Operating GHG emissions primarily result from the consumption of grid electricity (drawn from fossil fuel sources) when solar production is insufficient. The present value analysis has also been conducted for the operating GHG emissions during the entire 100-year life of the system. The discount rate for GHG emissions is taken as zero according to the suggestion by International Panel on Climate Change (IPCC) (Fearnside, 2002).

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#### 3.2. Hydraulic-power-based search space reduction method

In this paper, the power-based SSR method has been used to group pipes with similar hydraulic power capacities. The location, elevation, flow capacity, potential head loss and the pumping head have been considered in calculating the hydraulic power of a pipe (Zhao et al., 2023). Specifically, the maximum flow  $Q_{max}$  (based on five demand locading cases) and the corresponding residual pressure head  $p/\gamma$  at the outlet (from EPANET simulation with an assumed constant velocity) are estimated individually and multiplied to estimate the hydraulic power P of each pipe, which is given as

$$P = Q_{max}(p/\gamma) \qquad (3)$$

In this method, pipes with similar hydraulic power capacities are considered to have similar sizes. Then diameters of these pipes are put together to create a single decision variable to help reduce the search space size and improve optimisation efficiency.

#### 3.3. Discretisation of search space and performance evaluation measures

In order to explore to what extent the search space should be reduced or discretised for this optimisation problem, different numbers of decision variables have been considered when grouping pipe diameters based on their hydraulic power capacity using the power-based SSR method. According to different sizes of increments for hydraulic power values, 13 different numbers of decision variables are considered. The number of decision variables in each of the groups is 5, 10, 26, 50, 71, 94, 161, 197, 282, 318, 358, 370 and 432, respectively. For the last decision variable set, each individual pipe in this network is regarded as one decision variable. The selection of optimisation parameters for each decision variable set is based on common settings in the literature (Kollat and Reed, 2006; Wang et al., 2015; Wang et al., 2019). Furthermore, all optimisation runs were conducted on the Spartan High-Performance Computing (HPC) system based at the University of Melbourne, which integrates high-performance bare-metal computing with GPGPUs to cater to diverse needs. The Spartan HPC has 82 nodes with a combined total of 5904 cores (Intel(R) Xeon(R) Gold 6254 CPU @ 3.10 GHz) in a physical partition. Each node has a maximum RAM of approximately 1483 GB.

The hypervolume indicator (HI) as a measure for the convergence analysis used in multi-objective optimisation problems (Guerreiro et al., 2021) has been considered in this study. It calculates the area between a predetermined reference point (normally beyond the extreme values on the Pareto front) and all optimal solutions (Zitzler et al., 2003). A larger HI value implies a better convergence for this optimisation problem. In this study, two indicators have been evaluated to comprehensively understand the performance of the power-based SSR method when different levels of search space discretisation have been considered:

- 1) The number of evaluations needed to achieve convergence  $N_c$  has been evaluated for different numbers of decision variables. For each decision variable set, the HI value has been calculated for each generation for 10 optimisation runs starting with different random seeds, based on a predetermined reference point (300 (M\$) for LCC and 1500 (kt) for LCGHG). The speed of change (or the slope)  $m_A$  of the average of the 10 HI values against the increase in evaluations has been evaluated. When  $m_A \leq 0.01$ , it is regarded that convergence has been reached.
- 2) The HI values have been calculated for the final Pareto front in each decision variable set and compared, based on the same reference point as above.

# 4. **RESULTS AND DISCUSSION**

A total of 10 optimisation runs have been conducted for each decision variable set with different random seeds. Pareto fronts of final optimal solutions for each decision variable set are illustrated in Figure 3. The results of the two performance indicators against different numbers of decision variables are shown in Figure 4. The overall relationship between the two indicators is demonstrated in Figure 1 in the Abstract. The variation in the minimum LCC and the minimum LCGHG of final optimal solutions for each decision variable set is shown in Figure 5.

Trade-offs have been observed between the two objectives for different decision variable sets. As shown in Figure 3, an increase in the total life cycle cost can lead to a reduction in total life cycle GHG emissions. With the increase in the number of decision variables, better convergence has been achieved. When the number of decision variables is larger than 161, Pareto fronts are more clustered together with optimal solutions close to each other.



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Total life cycle cost (M\$)



Figure 3. Pareto fronts of different decision variable sets

Figure 4. (a) Number of evaluations needed to achieve convergence and (b) HI values of the final Pareto fronts for different decision variable sets



Figure 5. (a) The minimum total life cycle cost and (b) the minimum total life cycle GHG emissions of the final optimal solution for different decision variable sets

For the two performance indicators, first, the number of evaluations needed to achieve convergence  $N_c$  for each decision variable set is shown in Figure 4(a). Overall, with the increase in the number of decision variables, the number of evaluations needed to achieve convergence has increased. When the number of decision variables is larger than 282, the change in the number of evaluations required for convergence becomes insignificant. This indicates that when search space discretisation reaches a certain degree, optimisation efforts needed to converge become relatively stable.

Second, results for performance indicator 2 (Hypervolume Indicator (HI) value of the final Pareto front) for each decision variable set are shown in Figure 4(b). In general, an increase in the number of decision variables leads to a gradual increase in the HI values of final optimal solutions. When the number of decision variables is larger than 161, the change in the HI value becomes insignificant. This indicates that when search space discretisation reaches a certain point, further increasing the number of decision variables will not significantly improve the optimal solutions.

The general relationship between the two performance indicators is shown in Figure 1 in the Abstract. With an increase in the number of evaluations to achieve convergence (performance indicator 1), the Hypervolume Indicator (HI) value of final optimal solutions (performance indicator 2) increases. This implies that trade-offs between the speed of convergence and associated performance have been observed. Better performance is related to slower convergence. In addition, the minimum objective function values on each Pareto front for the final optimal solutions are illustrated in Figure 5. With an increase in the number of decision variables, both the minimum total life cycle cost and GHG emissions decrease. When the number of decision variables is larger than 161, the variation in both the minimum objective function values becomes insignificant. In real-world applications, decision-makers need to consider realistic computational resources and the accuracy of results required when discretising the search space using the power-based SSR method.

# 5. SUMMARY AND CONCLUSIONS

In this study, a real-world optimisation problem has been formulated to minimise both total life cycle costs and GHG emissions of a real-world pressurised irrigation system. A hydraulic-power-based search space reduction method (power-based SSR method) is used to improve the optimisation efficiency. The impact of search space discretisation on the optimisation performance has been investigated by considering different numbers of decision variables against two performance indicators: 1) the number of evaluations needed to achieve convergence; 2) Hypervolume Indicator (HI) values for the final Pareto front.

Results show that first, trade-offs between the two objective function values have been observed. With the increase in the number of decision variables, better convergence and smaller minimum objective function values can be achieved. Second, for the two performance indicators, an increase in the number of decision variables can lead to an increase in both the number of evaluations needed to achieve convergence and the HI value of final optimal solutions, when the number of decision variables is less than a certain threshold. In addition, there are also trade-offs observed between the two performance indicators (the speed of convergence and associated performance). Better convergence requires more optimisation effort, with an increased degree of search space discretisation. In real-world applications, both realistic computational resources and the required accuracy of results should be considered by decision-makers when discretising the search space using the power-based SSR method.

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