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THE EFFECT OF LEAD TIME ON BULLWHIP EFFECT IN SUPPLY CHAIN

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ABSTRACT

In supply chain management, one of the most critical problems which requires a lot of effort to deal with is the relief of bullwhip effect – the phenomenon in which information on demand is distorted while moving upstream. Although it is well established that demand forecast, lead time, order batching, shortage gaming and price fluctuation are the main sources that lead to the bullwhip effect, the problem of quantifying bullwhip effect is still remained unsolved in many situations due to the complex nature of the problem. In this research, a measure of bullwhip effect will be developed for a two-echelon system that includes one retailer and one supplier. The retailer employs base stock policy for their inventory and demand forecast is performed through autoregressive models. The effect of lead time will then be investigated.

1 INTRODUCTION

In supply chain, it has been observed that the variability in the ordering patterns usually increase when moving upstream towards manufacturers and suppliers. This phenomenon has first been observed and studied by some researchers (e.g., Forrester, 1958; Blinder, 1982,1986; Kahn, 1987). The Beer Game, developed at MIT and reported in the paper of Sterman (1989), which was widely used in teaching inventory management, is another demonstration of the above phenomenon.

The above phenomenon was first referred to as “bullwhip effect” in the works of Lee et al. (1997 a, b). Lee et al. (1997 a, b) has also proposed five important sources that might lead to the emergence of bullwhip effect in supply chains, i.e., demand signal processing, non-zero lead time, order batching, supply shortages and price fluctuations.

In order to avoid the bullwhip effect, all causes need to be eliminated. However, it is not so easy. Many approaches have been proposed to help alleviate the bullwhip effect. Berry et al. (1995) proposed redesign and reengineering of the supply chain as the tools to control demand amplification. Van Ackere et al. (1993) also recommended some approaches to avoid or alleviate the bullwhip effect, i.e., lead time reduction, information sharing or applying different replenishment rules for the inventory system.

Among the most important issues that need to be addressed in dealing with bullwhip effect, quantifying bullwhip effect is a very challenging issue. Graves (1999) quantified the bullwhip effect for the supply chain in which demand pattern follows an integrated moving average process. Chen et al. (2000 a, b) quantified the bullwhip effect for supply chains using moving average or exponential smoothing techniques for demand forecasts. In their works, it is assumed that members of the chain employ base stock policy for their inventory system. Dejonckheere et al. (2004) also investigated the base stock policy and it was found that under this replenishment policy, information sharing will help to significantly reduce the variance amplification of ordering quantities in supply chains. Although some researches have been conducted related to quantifying Bullwhip effect, more works need to be done in this field.

In this research, a measure of bullwhip effect will be developed for a two-echelon system that includes one retailer and one supplier. The retailer employs base stock policy for their inventory and demand forecast is performed through AR(1) autoregressive model. The effect of lead time on this measure will then be investigated.

The structure of this research is as follows. Section 2 discusses the AR(1) demand process and present the relationship between variance of demand and variance of error term in AR(1) demand process. The condition for the demand process to be a stationary one is also taken into consideration. Section 3 provides background on the determination of order quantity in base stock policy. In section 4, the expression for lead-time demand forecast with minimum expected mean squares of error is developed. Section 5 focuses on the determination of standard deviation of lead-time demand forecast error. Section 6 provides the expression for variance of order quantity and hence, the expression for the measure of bullwhip effect can be determined as the ration between variance of order quantity and variance of demand. The effects of autoregressive coefficient and lead time on bullwhip effect are considered in section 7 and some concluding remarks are discussed in section 8.

2 DEMAND PROCESS

<u>Notation</u>:

- D_t : Demand of period t
- q_t : Ordered quantity at the beginning of period t
- S_t : Order-up-to level at the beginning of period t
- ϕ : The first-order autocorrelation coefficient
- μ_d : Mean of the autoregressive process which is used to describe for demand process
- σ_d^2 : Variance of demand
- L : Order lead time
- D_t^L : Lead-time demand
- $\hat{D}_t^L$: Lead-time demand forecast
- $\hat{\sigma}_t^L$: Standard deviation of lead-time demand forecast error
- e_t : Forecast error for period t.

It is assumed that the demand can be modeled by an AR(1) model, in which

$$D_t = \delta + \phi D_{t-1} + \varepsilon_t \tag{1}$$

where ε_t is an i.i.d. normally distributed random variable with mean 0 and variance σ^2.

For the autoregressive process AR(1) to be stationary, we must have:

$$E[D_t] = E[D_{t-1}] = \mu_d \qquad \text{for all } t$$

therefore, it can be easily derived that

$$\mu_d = \frac{\delta}{1-\phi} \tag{2}$$

From (1), we also have: $\sigma_d^2 = \phi^2 \sigma_d^2 + \sigma^2$

Hence,

$$\sigma_d^2 = \frac{\sigma^2}{1-\phi^2} \tag{3}$$

From (2) and (3), it can be seen that in order for the process to be stationary we should also have $|\phi| < 1$.

3 ORDER QUANTITY IN BASE STOCK POLICY

Let S_t be the order-up-to level at the beginning of period *t*, i.e., the inventory position at the beginning of period *t* after the order has been placed, the order quantity at the beginning of period *t* should satisfy the following relationship:

$$q_t = S_t - S_{t-1} + D_{t-1} \tag{4}$$

If the base stock policy is employed, the order-up-to level S_t can be determined through lead-time demand by:

$$S_t = \hat{D}_t^L + z\hat{\sigma}_t^L \tag{5}$$

in which *z* is the normal *z*-score which can be determined based on the desired service level of the inventory policy.

It should be noted that the optimal order-up-to level S_t can be explicitly determined from inventory holding cost and shortage cost (e.g., Heyman and Sobel, 1984). However, it is usually difficult to accurately estimate these costs in practice, the approach using service level, which is defined as the probability for the on-hand inventory fulfilling lead-time demand requirement, to help determine order-up-to level are usually more practicable.

It is noted that the base stock policy is a special case of the order-up-to (s, S) inventory policy in which $s = S - 1$. In the base stock policy, an order will be placed at the beginning of each period so as to increase the inventory level up to a predefined level *S*. In general, the order-up-to policy is an optimal inventory policy in systems where there is no fix ordering cost and both holding and shortage costs are proportional to the volume of on-hand inventory or shortage (Nahmias, 1997; Zipkin, 2000).

At a predefined service level, the determination of lead-time demand forecast $\hat{D}_t^L$ and standard deviation of lead-time demand forecast error $\hat{\sigma}_t^L$ is needed in order to help determine the order-up-to level S_t. The expressions for calculation of $\hat{D}_t^L$ and $\hat{\sigma}_t^L$ are developed in section 4 and section 5, respectively. However, as will be shown later, $\hat{\sigma}_t^L$ is not depend on *t* and hence, this value does not have any influence on bullwhip effect.

4 DETERMINATION OF LEAD-TIME DEMAND FORECAST

In this research, it is assumed that demand forecast should be conducted so as to minimize the expected mean squares of error. It is noted that lead-time demand can be expressed as:

$$D_t^L = D_t + D_{t+1} + ... + D_{t+L-1} = \sum_{i=0}^{L-1} D_{t+i} \quad (6)$$

If $\hat{D}_t$ is the forecast with minimum expected mean squares of error of D_t, then the minimum expected mean squares of error forecast for lead-time demand can be determined as

$$\hat{D}_t^L = \hat{D}_t + \hat{D}_{t+1} + ... + \hat{D}_{t+L-1} = \sum_{i=0}^{L-1} \hat{D}_{t+i} \quad (7)$$

For the AR(1) process, $\hat{D}_t$ can be determined by (Pindyck and Rubinfeld, 1998):

$$\hat{D}_{t+i} = E\left[D_{t+i} \,|\, D_{t-1}, D_{t-2}, ...\right]$$

From the above expression, the explicit expression of $\hat{D}_t$ can be derived and it has the following form:

$$\hat{D}_{t+i} = \mu_d \left(1-\phi^{i+1}\right) + \phi^{i+1} D_{t-1} \quad (8)$$

Proof:

We have: $D_{t+i} = \delta + \phi D_{t+i-1} + \varepsilon_{t+i}$

If we express D_{t+i-1} in terms of D_{t+i-2}:

$$D_{t+i-1} = \delta + \phi D_{t+i-2} + \varepsilon_{t+i-1}$$

Then $D_{t+i} = \delta\left(1+\phi\right) + \phi^2 D_{t+i-2} + \varepsilon_{t+i} + \phi\varepsilon_{t+i-1}$

Applying the above procedure successively, we have:

$$D_{t+i} = \delta\left(1+\phi+...+\phi^i\right) + \phi^{i+1} D_{t-1} + \varepsilon_{t+i} + \phi\varepsilon_{t+i-1} + ...\phi^i \varepsilon_t$$

Taking the expectation we have:

$$\begin{aligned}\hat{D}_{t+i} &= E\left[D_{t+i} \,|\, D_{t-1}, D_{t-2}, ...\right] \\ &= \delta\left(1+\phi+...+\phi^i\right) + \phi^{i+1} D_{t-1} \\ &= \delta \frac{1-\phi^{i+1}}{1-\phi} + \phi^{i+1} D_{t-1} \\ &= \mu_d \left(1-\phi^{i+1}\right) + \phi^{i+1} D_{t-1}\end{aligned}$$

Proposition 1: The forecast of lead-time demand with minimum expected mean squares of error can be determined by:

$$\hat{D}_t^L = \mu_d \left(L - \frac{\phi\left(1-\phi^L\right)}{1-\phi}\right) + \frac{\phi\left(1-\phi^L\right)}{1-\phi} D_{t-1} \quad (9)$$

Proof:

Replacing (8) into (7), we have:

$$\begin{aligned}\hat{D}_t^L &= \sum_{i=0}^{L-1}\left\{\mu_d\left(1-\phi^{i+1}\right) + \phi^{i+1} D_{t-1}\right\} \\ &= \mu_d \sum_{i=0}^{L-1}\left(1-\phi^{i+1}\right) + D_{t-1}\sum_{i=0}^{L-1}\phi^{i+1} \\ &= \mu_d \left(L - \frac{\phi\left(1-\phi^L\right)}{1-\phi}\right) + \frac{\phi\left(1-\phi^L\right)}{1-\phi} D_{t-1}\end{aligned}$$

5 DETERMINATION OF STANDARD DEVIATION OF LEAD-TIME DEMAND FORECAST ERROR

In this section, the expression of variance of lead-time demand forecast error will be developed.

Proposition 2: The variance of lead-time demand forecast error is not depend on *t* and determined by:

$$\left(\hat{\sigma}_t^L\right)^2 = \frac{\sigma_d^2\left(1+\phi\right)}{1-\phi}\sum_{i=1}^{L}\left(1-\phi^i\right)^2 \quad (10)$$

Proof:

By definition we have: $\left(\hat{\sigma}_t^L\right)^2 = VAR\left(D_t^L - \hat{D}_t^L\right)$

We also have:

$$\begin{aligned}D_t^L - \hat{D}_t^L &= \left(D_t - \hat{D}_t\right) + \left(D_{t+1} - \hat{D}_{t+1}\right) + ... + \left(D_{t+L-1} - \hat{D}_{t+L-1}\right) \\ &= e_t + e_{t+1} + e_{t+L-1} = \sum_{i=0}^{L-1} e_{t+i}\end{aligned}$$

From the proof of expression (8), it can be seen that

$$\begin{aligned}e_{t+i} &= D_{t+i} - \hat{D}_{t+i} = \varepsilon_{t+i} + \phi\varepsilon_{t+i-1} + ...\phi^i\varepsilon_t \\ &= \sum_{j=o}^{i}\varepsilon_{t+j}\phi^{i-j}\end{aligned}$$

Hence,

$$D_t^L - \hat{D}_t^L = \sum_{i=0}^{L-1}\sum_{j=0}^{i}\varepsilon_{t+j}\phi^{i-j} = \sum_{i=0}^{L-1}\sum_{j=0}^{L-i-1}\varepsilon_{t+i}\phi^j = \sum_{i=0}^{L-1}\varepsilon_{t+i}\frac{1-\phi^{L-i}}{1-\phi}$$

Therefore,

$$\begin{aligned}&VAR\left(D_t^L - \hat{D}_t^L\right) \\ &= \sum_{i=0}^{L-1}\left(\frac{1-\phi^{L-i}}{1-\phi}\right)^2\sigma^2 = \frac{\sigma^2}{\left(1-\phi\right)^2}\sum_{i=0}^{L-1}\left(1-\phi^{L-i}\right)^2 \\ &= \frac{\sigma_d^2\left(1+\phi\right)}{\left(1-\phi\right)}\sum_{i=1}^{L}\left(1-\phi^i\right)^2\end{aligned}$$

It should be noted that $\hat{\sigma}_t^L$ is needed for the determination of the order-up-to level S_t only. The measure of bullwhip effect developed in this research is not affected by this value.

6 DETERMINATION OF VARIANCE OF ORDER QUANTITY

Proposition 3: The variance of order quantity of period t can be determined by the following expression

$$VAR(q_t) = \frac{(1+\phi)\left(1-2\phi^{L+1}\right)+2\phi^{2L+2}}{1-\phi}\sigma_d^2 \tag{11}$$

Proof:

We have:
$$\begin{aligned} q_t &= S_t - S_{t-1} + D_{t-1} \\ &= \left(\hat{D}_t^L + z\hat{\sigma}_t^L\right) - \left(\hat{D}_{t-1}^L + z\hat{\sigma}_{t-1}^L\right) + D_{t-1} \\ &= \left(\hat{D}_t^L - \hat{D}_{t-1}^L\right) + D_{t-1} \end{aligned}$$

From (9), it can be easily derived that

$$q_t = \frac{1-\phi^{L+1}}{1-\phi}D_{t-1} - \frac{\phi\left(1-\phi^L\right)}{1-\phi}D_{t-2}$$

So,

$$VAR(q_t) = \left(\frac{1-\phi^{L+1}}{1-\phi}\right)^2 VAR(D_{t-1}) + \frac{\phi^2\left(1-\phi^L\right)^2}{(1-\phi)^2}VAR(D_{t-2})$$
$$-2\frac{1-\phi^{L+1}}{1-\phi}\frac{\phi\left(1-\phi^L\right)}{1-\phi}COV\left(D_{t-1},D_{t-2}\right)$$

It is noted that
$$VAR(D_{t-1}) = VAR(D_{t-2}) = \sigma_d^2$$
$$COV\left(D_{t-1},D_{t-2}\right) = \phi\sigma_d^2$$

Therefore,

$$\begin{aligned} &VAR(q_t) \\ &= \left(\frac{1-\phi^{L+1}}{1-\phi}\right)^2\sigma_d^2 + \frac{\phi^2\left(1-\phi^L\right)^2}{(1-\phi)^2}\sigma_d^2 - 2\frac{1-\phi^{L+1}}{1-\phi}\frac{\phi\left(1-\phi^L\right)}{1-\phi}\phi\sigma_d^2 \\ &= \frac{\left(1-\phi^{L+1}\right)^2 + \phi^2\left(1-\phi^L\right)^2 - 2\phi^2\left(1-\phi^{L+1}\right)\left(1-\phi^L\right)}{(1-\phi)^2}\sigma_d^2 \\ &= \frac{(1+\phi) - 2\phi^{L+1}(1+\phi) + 2\phi^{2L+2}}{1-\phi}\sigma_d^2 \\ &= \frac{(1+\phi)\left(1-2\phi^{L+1}\right)+2\phi^{2L+2}}{1-\phi}\sigma_d^2 \end{aligned}$$

From (11), the measurement of bullwhip effect can be determined as:

$$B(L,\phi) = \frac{VAR(q_t)}{\sigma_d^2} = \frac{(1+\phi)\left(1-2\phi^{L+1}\right)+2\phi^{2L+2}}{1-\phi} \tag{12}$$

7 EFFECTS OF AUTOREGRESSIVE COEFFICIENT AND LEAD TIME

7.1 The Effect of Autoregressive Coefficient on Bullwhip Effect

In this section, we first investigate the effect of autoregressive coefficient ϕ on the measure of bullwhip effect. From (12), we can see that

$$\begin{aligned} B(L,\phi) \le 1 \quad &\Leftrightarrow \quad \frac{(1+\phi)\left(1-2\phi^{L+1}\right)+2\phi^{2L+2}}{1-\phi} \le 1 \\ &\Leftrightarrow \quad (1+\phi)\left(1-2\phi^{L+1}\right)+2\phi^{2L+2} \le 1-\phi \\ &\Leftrightarrow \quad 2\phi\left(1-\phi^L\right)\left(1-\phi^{L+1}\right) \le 0 \\ &\Leftrightarrow \quad \phi \le 0 \end{aligned}$$

This means that the bullwhip effect occurs only when there exists a positive autoregressive relationship in the demand process. Due to this fact, we consider only the case when $0 < \phi < 1$ in the remaining parts of this paper.

In order to detect the effect of autoregressive coefficient, it is noted that the measure of bullwhip effect $B(L,\phi)$ developed above can be expressed as

$$B(L,\phi) = 1 + \sum_{i=1}^{L} f_i(L,\phi) \tag{13}$$

in which $f_i(L,\phi) = 2\phi^i\left(1-\phi^{L+1}\right) \quad (i = 1,2,...,L)$. In fact, the above expression can be developed as follows:

$$\begin{aligned} B(L,\phi) &= \frac{(1+\phi)\left(1-2\phi^{L+1}\right)+2\phi^{2L+2}}{1-\phi} \\ &= \frac{1+\phi-2\phi^{L+1}-2\phi^{L+2}\left(1-\phi^L\right)}{1-\phi} \\ &= \frac{(1-\phi)\left(1+2\phi+2\phi^2+...+2\phi^L\right) - 2\phi^{L+2}(1-\phi)\left(1+\phi+\phi^2+..+\phi^{L-1}\right)}{1-\phi} \\ &= \left(1+2\phi+2\phi^2+...+2\phi^L\right) - 2\phi^{L+2}\left(1+\phi+\phi^2+..+\phi^{L-1}\right) \\ &= 1 + \sum_{i=1}^{L} 2\phi^i\left(1-\phi^{L+1}\right) \end{aligned}$$

Proposition 4: In the interval $0<\phi<1$, the functions $f_i(L,\phi);\ (i=1,2,...,L)$ are increasing and then decreasing. The maximum values are reached at

$$\phi_i^* = \left(\frac{i}{L+i+1}\right)^{1/L+1}$$

Moreover, ϕ_i^* is an increasing function with respect to *i*, $(i=1,2,...,L)$.

Proof:

Taking the first derivative of $f_i(L,\phi)=2\phi^i\left(1-\phi^{L+1}\right)$ with respect to ϕ, we have:

$$\frac{\partial f_i(L,\phi)}{\partial\phi} = 2\left[i\phi^{i-1}-(L+i+1)\phi^{L+i}\right] = 2\phi^{i-1}\left[i-(L+i+1)\phi^{L+1}\right]$$

Solving the equation: $\frac{\partial f_i(L,\phi)}{\partial\phi}=0$, we have:

$$\phi_i^* = \left(\frac{i}{L+i+1}\right)^{1/L+1}$$

In the interval $\left(0,\phi_i^*\right)$, $\frac{\partial f_i(L,\phi)}{\partial\phi}>0$ and in the interval $\left(\phi_i^*,1\right)$, $\frac{\partial f_i(L,\phi)}{\partial\phi}<0$. Hence, $f_i(L,\phi)$ is an increasing and then decreasing function.

It is also noted that $\frac{i}{L+i+1}<1$. Therefore, ϕ_i^* is an increasing function with respect to *i*, $(i=1,2,...,L)$.

In the next session, the behavior of $B(L,\phi)$ with respect to ϕ will be investigates.

Proposition 5: The measure of bullwhip effect $B(L,\phi)$ posses the following properties:

a. $B(L,\phi)$ is an increasing function in the interval $\phi\in\left(0,\phi_1^*\right)$, in which $\phi_1^*=\left(\frac{1}{L+2}\right)^{1/L+1}$

b. $B(L,\phi)$ is a concave function in the interval $\phi\in\left(\phi_1^*,1\right)$.

Proof:

a. Due to the fact that $f_i(L,\phi);\ (i=1,2,...,L)$ are increasing functions in $\left(0,\phi_i^*\right)$ and that ϕ_i^* is an increasing function with respect to *i*, $(i=1,2,...,L)$, the first statement of proposition 5 is easily to be confirmed.

b. In order to prove that $B(L,\phi)$ is a concave function in the interval $\phi\in\left(\phi_1^*,1\right)$, we first prove that $f_i(L,\phi);\ (i=1,2,...,L)$ are concave in $\phi\in\left(\phi_1^*,1\right)$. Consider the second-order derivative of $f_i(L,\phi)$ with respect to ϕ, we have:

$$\frac{\partial^2 f_i(L,\phi)}{\partial\phi^2} = 2\left[i(i-1)\phi^{i-2}-(L+i+1)(L+i)\phi^{L+i-1}\right] = 2\phi^{i-2}\left[i(i-1)-(L+i+1)(L+i)\phi^{L+1}\right]$$

We need to prove that: $\frac{\partial^2 f_i(L,\phi)}{\partial\phi^2}<0$ when $\phi>\phi_1^*=\left(\frac{1}{L+2}\right)^{1/L+1}$.

Note that: $\frac{\partial^2 f_i(L,\phi)}{\partial\phi^2}<0 \Leftrightarrow \phi^{L+1}>\frac{i(i-1)}{(L+i+1)(L+i)}$

The above inequality is hold true due to the fact that:

$$\phi^{L+1}>\frac{1}{L+2}>\frac{i}{L+i+1}>\frac{i(i-1)}{(L+i+1)(L+i)}.$$

It is noted that the sum of concave functions is also a concave function. Hence, $B(L,\phi)$, which is the sum of $f_i(L,\phi);\ (i=1,2,...,L)$ and a constant, is a concave function in the interval $\phi\in\left(\phi_1^*,1\right)$.

From proposition 5, it can be concluded that there always exist a unique value of $\phi_{\text{opt}}\in\left(\phi_1^*,1\right)$ such that $B(L,\phi)$ reaches its maximum value at a fixed value of lead time *L*. Furthermore, from the results in proposition 4, it can be easily seen that $\phi_{\text{opt}}\in\left(\phi_1^*,\phi_L^*\right)$ and hence, bisection method can be applied in this interval to find ϕ_{opt}.

The behavior of $B(L,\phi)$ with respect to ϕ is shown in Figure 1 below:

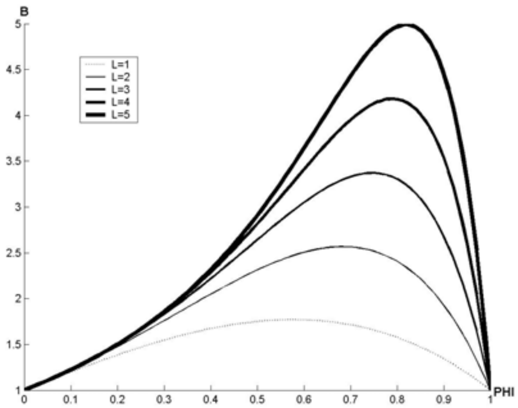

Figure 1: Effect of Autoregressive Coefficient on Bullwhip Measure

7.2 The Effect of Lead Time on Bullwhip Effect

In this section, we will investigate the effect of lead time on the measure of bullwhip effect. Taking the first derivative of $B(L,\phi)$ with respect to L we have

$$\frac{\partial B}{\partial L} = \frac{-2(1+\phi)\phi^{L+1}\ln\phi + 4\phi^{2L+2}\ln\phi}{1-\phi}$$

With the condition $|\phi| < 1$, it is easily to prove that $\frac{\partial B}{\partial L} > 0$. Therefore, $B(L,\phi)$ is an increasing function with respect to *L*. It can be concluded that the longer the lead time, the larger the bullwhip effect in the supply chain. This finding is not contradictory to intuition and figure 1 above also show this aspect.

From expression (12), we can also prove that $B(L,\phi) < \frac{1+\phi}{1-\phi}$. This upper bound can be found by taking the limitation of $B(L,\phi)$ when *L* approaches infinity. Although the maximum value of bullwhip measure can be accurately determined for any value of *L*, the above upper bound value can be used as a rough estimator for the maximum value of bullwhip measure when lead time increases.

The functional expression of bullwhip effect represented by (12) also exhibit an interesting property stated in the following proposition

Proposition 6: Consider values of ϕ in the interval $(0,1)$

a. If $\phi \le 1/3$ then $B(L,\phi)$ is a concave function with respect to *L*

b. If $\phi > 1/3$ then $B(L,\phi)$ is a convex and then concave function with respect to *L*.

Proof:
Consider the second derivative of $B(L,\phi)$ with respect to *L*, we have

$$\frac{\partial^2 B}{\partial L^2} = \frac{2(\ln\phi)^2\phi^{L+1}(4\phi^{L+1}-\phi-1)}{1-\phi}$$

a. If $\phi \le 1/3$: $(4\phi^{L+1}-\phi-1) \le 0$ when $L=0$. Furthermore, when *L* increases ϕ^{L+1} will decrease. Therefore, $(4\phi^{L+1}-\phi-1) \le 0$ for all values $L \ge 0$. This leads to the fact that $\frac{\partial^2 B}{\partial L^2} \le 0 \quad \forall L \ge 0$ and hence, $B(L,\phi)$ is a concave function with respect to *L*.

b. If $\phi > 1/3$: $(4\phi^{L+1}-\phi-1) > 0$ when $L=0$. However, when *L* increases ϕ^{L+1} will decrease and starting at $L^* = \log_\phi\left(\frac{1+\phi}{4}\right) - 1$, $(4\phi^{L+1}-\phi-1)$ will receive negative values. This means that $\frac{\partial^2 B}{\partial L^2}$ will first have positive values and then receive negative values when *L* increases. Therefore, $B(L,\phi)$ is a convex and then concave function with respect to *L*.

The above properties are illustrated in Figures 2a and 2b for the cases $\phi = 0.3$ and $\phi = 0.9$.

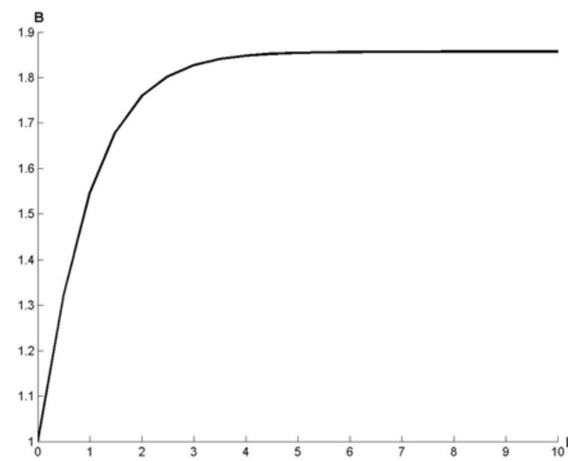

Figure 2a: Effect of Lead Time on Bullwhip Measure ($\phi = 0.3$)

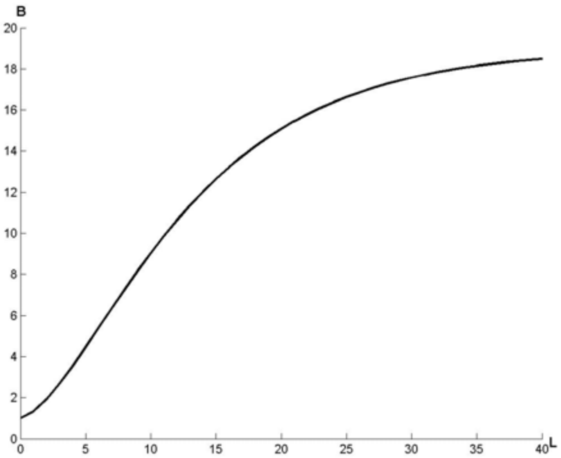

Figure 2b: Effect of Lead Time on Bullwhip Measure ($\phi = 0.9$)

8 CONCLUSIONS

In this research, we have investigated a simple supply chain that includes one retailer and one supplier in which the retailer employs base stock inventory policy. The bullwhip measure is developed for AR(1) demand process and some interesting properties of this measure have been found.

Firstly, it is interesting to note that the bullwhip effect does not exist not only when there is no correlation or there is negative correlation in the demand process but also when there exists a perfect positive correlation in the demand process (i.e., $\phi = 1$). In the range $\phi \in (0,1)$, when ϕ increases, the bullwhip effect will first increases, reaches maximum value, and then decreases. It is also noted that when the lead time increases, the value of ϕ at which the bullwhip effect reaches its maximum value also increases and approaches one.

Another interested finding of this research is that there exists an upper bound for the measure of bullwhip effect. The value of this upper bound depends on the autoregressive coefficient ϕ. Moreover, it should be noted that the behavior of the bullwhip measure can have two patterns with respect to lead time. This also depends on the autoregressive coefficient ϕ.

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