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SPILLOVER EFFECTS IN FORECASTING VOLATILITY AND VALUE-AT-RISK

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ABSTRACT

Accurate modelling of volatility (or risk) is important in finance, particularly as it relates to the modelling and forecasting of Value-at-Risk (VaR) thresholds. As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past, as well as the past of other assets. In this paper we analyse the importance of considering spillover effects when forecasting financial volatility. The forecasting performance of the VARMA-GARCH model of Ling and McAleer (2003), which includes spillover effects from all assets, the CCC model of Bollerslev (1990), which includes no such spillovers, and the new PS-GARCH model, which accommodates aggregate spillovers parsimoniously, are compared using a VaR example. The empirical results suggest that the inclusion of spillover effects is not necessarily important in forecasting VaR thresholds, even when these volatility spillovers are statistically significant.

1. INTRODUCTION

Accurate modelling of volatility (or risk) is of paramount importance in finance. As risk is unobservable, several modelling procedures have been developed to measure and forecast risk. The Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986) has subsequently led to a family of autoregressive conditional volatility models. The success of GARCH models can be attributed largely to their ability to capture several stylised facts of financial returns, such as time-varying volatility, persistence and clustering of volatility, and asymmetric reactions of risk to positive and negative shocks of equal magnitude. This has also contributed to the modelling, forecasting and backtesting of Value-at-Risk (VaR) thresholds.

As financial applications typically deal with a portfolio of assets and risks, there are several multivariate GARCH models which specify the risk of one asset as depending dynamically on its own past risk as well as on the past risk of other assets (see McAleer (2005) for a discussion of a variety of univariate and multivariate conditional and stochastic volatility models). da Veiga and McAleer (2005) showed that the VARMA-GARCH model of Ling and McAleer (2003) and the VARMA-AGARCH model of Hoti et al. (2003) provided far superior volatility estimates and VaR threshold forecasts than their nested univariate counterparts, namely the GARCH model of Bollerslev (1986) and the GJR model of Glosten, Jagannathan and Runkle (1992), respectively.

In this paper we investigate the importance of including spillover effects in modelling and forecasting financial volatility and VaR thresholds. We compare the forecasted conditional variances produced by the VARMA-GARCH model of Ling and McAleer (2003), in which the conditional variance of asset i is specified to depend dynamically on past squared unconditional shocks and past conditional variances of each asset in the portfolio, with the forecasted conditional variances produced by the CCC model of Bollerslev (1990), where the conditional variance of asset i is specified to depend only on the squared unconditional shocks and past conditional variances of asset i. We also develop a new Portfolio Spillover GARCH (PS-GARCH) model, which allows spillover effects to be included parsimoniously. This parsimonious and computationally straightforward model is found to yield volatility estimates and VaR threshold forecasts that are very similar to those of the VARMA-GARCH and VARMA-AGARCH models.

The plan of the paper is as follows. Section 2 presents the new portfolio spillover GARCH (PS-GARCH) model, discusses alternative multivariate GARCH models with and without spillover effects, and presents a simple two-step estimation method for PS-GARCH. The data are discussed in Section 3, forecasting is examined in Section 4, the economic significance of the results is analysed in Section 5, and some concluding remarks are given in Section 6.

MODELS AND ESTIMATION

This section proposes a parsimonious PS-GARCH model which captures aggregate portfolio spillover effects, and compares the new model with two constant conditional correlation models, one of which models spillover effects from each of the other assets in the portfolio and another which has no portfolio spillover effects.

2.1 PS-GARCH

Let the vector of returns on *m* financial assets be given by

$$Y_t = E(Y_t / F_{t-1}) + \varepsilon_t \tag{1}$$

where the conditional mean of the returns follows a VARMA process:

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \tag{2}$$

The return on the portfolio consisting of the *m* assets is denoted as:

$$Y_{p,t} = E(\sum_{i=1}^{m} x_{i,t} y_{i,t} / F_{t-1}) + \varepsilon_{p,t} \tag{3}$$

where $y_{i,t}$ denotes the return on asset *i* at time *t* and x_{it} denotes the portfolio weight of asset *i* at time *t*, such that:

$$\sum_{i=1}^{m} x_{i,t} = 1 \quad \forall\, t . \tag{4}$$

The portfolio spillover GARCH (PS-GARCH) model assumes that the returns on the portfolio follow an ARMA process, as follows:

$$\Phi(L)(Y_{p,t} - \mu_p) = \Psi(L)\varepsilon_{p,t} \tag{5}$$

$$\varepsilon_t = D_t \eta_t \tag{6}$$

$$\varepsilon_{p,t} = h_{p,t}^{1/2} \eta_{p,t} \tag{7}$$

$$h_{p,t} = \omega_p + \sum_{l=1}^{s} \alpha_{p,l} \varepsilon_{p,t-l}^2 + \sum_{l=1}^{s} \beta_{p,l} h_{p,t-l} \tag{8}$$

$$H_t = W + \sum_{l=1}^{r} A_l \varepsilon'_{t-l} + \sum_{l=1}^{r} C_l I(\eta_{t-l}) \varepsilon'_{t-l} + \sum_{l=1}^{s} B_l H_{t-l} + \sum_{l=1}^{s} G_l \hat{\varepsilon}_{p,t-l} + \sum_{l=1}^{s} K_l \hat{h}_{p,t-l} \tag{9}$$

where $H_t = (h_{1t},..., h_{mt})'$, $W = (\omega_1,..., \omega_m)'$, $D_t = diag(h_{it}^{1/2})$, $\eta'_t = (\eta_{1t},...,\eta_{mt})$, $\varepsilon'_t = (\varepsilon_{1t}^2,...,\varepsilon_{mt}^2)$, and $\hat{\varepsilon}_{p,t-l}^2$ and $\hat{h}_{p,t-l}$ are the fitted values from equations (5) and (6), respectively. The matrices A_l, B_l, C_l, G_l and K_l are diagonal, with typical elements given by α_{ii}, β_{ii}, γ_{ii}, λ_{ii} and δ_{ii}, respectively, for $i = 1,...,m$, $I(\eta_t) = diag(I(\eta_{it}))$ is an $m \times m$ diagonal matrix of indicator functions, the operators $\Phi(L) = I_m - \Phi_1 L - ... - \Phi_p L^p$ and $\Psi(L) = I_m - \Psi_1 L - ... - \Psi_q L^q$ are polynomials in *L*, the lag operator, F_t is the past information available to time *t*, I_m is the $m \times m$ identity matrix, and $I(\eta_{it})$ is an indicator function, given as: (0.1)

$$I(\eta_{it}) = \begin{cases} 1, \ \varepsilon_{it} \le 0 \\ 0, \ \varepsilon_{it} > 0. \end{cases} \tag{10}$$

The indicator function distinguishes between the effects of positive and negative shocks of equal magnitude on conditional volatility.

Using (6), the conditional covariance matrix for the PS-GARCH model is given by $Q_t = D_t \Gamma D_t$, for which the matrix of conditional correlations is given by $E(\eta_t \eta'_t) = \Gamma$. The matrix Γ is the constant conditional correlation matrix of the unconditional shocks which is, by definition, equivalent to the constant conditional correlation matrix of the conditional shocks. It is possible to relax this assumption to make Γ vary over time.

2.2 VARMA-GARCH

The VARMA-GARCH model of Ling and McAleer (2003), which assumes symmetry in the effects of positive and negative shocks on conditional volatility, is given by:

$$Y_t = E(Y_t / F_{t-1}) + \varepsilon_t \qquad (11)$$

$$\Phi(L)(Y_t - \mu) = \Psi(L)\varepsilon_t \qquad (12)$$

$$\varepsilon_t = D_t \eta_t \qquad (13)$$

$$H_t = W + \sum_{l=1}^{r} A_l \vec{\varepsilon}_{t-l} + \sum_{l=1}^{s} B_l H_{t-l} \qquad (14)$$

where $H_t = (h_{it},...,h_{mt})'$, $W = (\omega_1,...,\omega_m)'$, $D_t = diag(h_{i,t}^{1/2})$, $\eta_t = (\eta_{i,t},...,\eta_{mt})'$, $\vec{\varepsilon}_t = (\varepsilon_{it}^2,...,\varepsilon_{mt}^2)$, A_l and B_l are $m \times m$ matrices with typical elements α_{ij} and β_{ij}, respectively, for $i, j = 1,...,m$, $I(\eta_t) = diag(I(\eta_{it}))$ is an $m \times m$ matrix, $\Phi(L) = I_m - \Phi_1 L - ... - \Phi_p L^p$ and $\Psi(L) = I_m - \Psi_1 L - ... - \Psi_q L^q$ are polynomials in L, the lag operator, and F_t is the past information available to time t. Spillover effects are given in the conditional volatility for each asset in the portfolio. Based on equation (13), the VARMA-GARCH model also assumes that the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$.

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An extension of the VARMA-GARCH model is the VARMA-AGARCH model of Hoti et al. (2002), which captures the asymmetric spillover effects from each of the other assets in the portfolio. The VARMA-AGARCH model is also a multivariate extension of the univariate GJR model.

2.3 CCC

The VARMA-GARCH and VARMA-AGARCH models have several popular constant conditional correlation univariate and multivariate models as special cases. If the model given by equation (14) is restricted so that A_l and B_l are diagonal matrices, the VARMA-GARCH model reduces to:

$$h_{it} = \omega_i + \sum_{l=1}^{r} \alpha_i \varepsilon_{i,t-l} + \sum_{l=1}^{s} \beta_i h_{i,t-l} \qquad (15)$$

which is the constant conditional correlation (CCC) model of Bollerslev (1990). The CCC model also assumes that the matrix of conditional correlations is given by $E(\eta_t \eta_t') = \Gamma$. As given in equation (15), the CCC model does not have volatility spillover effects across different financial assets, and hence is intrinsically univariate in nature. Moreover, CCC also does not capture the asymmetric effects of positive and negative shocks on conditional volatility.

3. DATA

The data used in the empirical application are daily prices measured at 16:00 Greenwich Mean Time (GMT) for four international stock market indices (henceforth referred to as synchronous data), namely S&P500 (USA), FTSE100 (UK), CAC40 (France), and SMI (Switzerland). All prices are expressed in US dollars. The data were obtained from DataStream for the period 3 August 1990 to 5 November 2004. At the time the data were collected, this period was the longest for which data on all four variables were available. The rationale for employing daily synchronous data in modelling stock returns and volatility transmission is four-fold.

First, the Efficient Markets Hypothesis would suggest that information is quickly and efficiently incorporated into stock prices. While information generated yesterday may be significant in explaining stock price changes today, it is less likely that news generated last month would have any explanatory power today.

Second, it has been argued by Engle et al. (1990) that volatility is caused by the arrival of unexpected news, so that volatility clustering is the result of investors reacting differently to news. The use of daily data may help to model interactions between the heterogeneity of investor responses in different markets.

Third, studies that use close-to-close non-synchronous returns suffer from the non-synchronicity problem, as highlighted in Scholes and Williams (1977). In particular, these studies cannot distinguish a spillover from a contemporaneous correlation when markets with common trading hours are analysed. Kahya (1997) and Burns et al. (1998) also observe that, if cross market correlations are positive, the use of close-to-close returns for non-synchronous markets will underestimate the true correlations, and hence underestimate the true risk associated with a portfolio of such assets.

Finally, the use of synchronous data allows the system to be written in a simultaneous equations form, which can be estimated jointly. Such joint estimation of the parameters eliminates potential econometric problems associated with generated regressors (see, for example, Pagan (1984) and Oxley and McAleer (1993, 1994)), improves efficiency in estimation, increases the power of the test for cross-market spillovers, and analyses market interactions simultaneously. This allows all the relationships to be tested jointly. Joint estimation is also consistent with the notion that spillovers can capture the impact of global news on each market.

The synchronous returns for each market i at time t, $(R_{i,t})$, are defined as:

$$R_{i,t} = \log(P_{i,t} / P_{i,t-1}),$$

where $P_{i,t}$ is the price in market i at time t, as recorded at 16:00 GMT.

The descriptive statistics for the synchronous returns of the four indexes are given in Table 1. All series have similar means and medians at close to zero, minima which vary between -5.533 and -10.251, and maxima that range between 5.771 and 10.356. Although the four standard deviations vary slightly, the coefficients of variation (CoV) are quite different, ranging from 31.227 for S&P500 to 66.002 for CAC40. The skewness differs among all four series, but the kurtosis is similar for all series. The Jarque-Bera test of normality strongly rejects the null hypothesis of normally distributed returns, which may be due to the presence of extreme observations. As each of the series displays a high degree of kurtosis, this would seem to indicate the existence of extreme observations.

4. FORECASTS

The aim of this section is to compare the volatility and conditional correlation forecasts produced by the CCC model of Bollerslev (1990), the VARMA-GARCH model of Ling and McAleer (2003), and the PS-GARCH model proposed in this paper. We use a rolling window approach to forecast the 1-day ahead conditional correlations and conditional variances. The sample ranges from 3 August 1990 to 5 November 2004. In order to strike a balance between efficiency in estimation and a viable number of rolling regressions, the rolling window size is set at 2000 for all four data sets, which leads to a forecasting period from 6 April 1998 to 5 November 2004.

Figures 1-4 plot the forecasted volatilities for each returns series using the 3 models. The volatility forecasts produced by all models are remarkably similar, with correlation coefficients of the volatility forecasts ranging from 0.955 to 0.990, suggesting that PS-GARCH provides a convenient and parsimonious approximation to the VARMA-GARCH model.

5. ECONOMIC SIGNIFICANCE

The 1988 Basel Capital Accord, which was originally concluded between the central banks from the Group of Ten (G10) countries, and has since been adopted by over 100 countries, sets minimum capital requirements which must be met by banks to guard against credit and market risks. These capital requirements are a function of the forecasted VaR thresholds, in which VaR summarizes the maximum expected loss over a target horizon for a given level of confidence. The Basel Accord stipulates that the daily capital charge must be set at the higher of the previous day’s VaR or the average VaR over the last 60 business days, multiplied by a factor *k.* The multiplicative factor *k* is set by the local regulators, but must not be lower than 3. In 1995, the 1988 Basel Accord was amended to allow banks to use internal models to determine their VaR. However, banks wishing to use internal models must demonstrate that the models are sound. Furthermore, the Basel Accord imposes penalties in the form of a higher multiplicative factor *k* on banks which use models that lead to a greater number of violations than would reasonably be expected given the specified confidence level of 1%. Table 2 shows the penalties imposed for a given number of violations for 250 business days.

In certain cases, where the number of violations is deemed to be excessively large, regulators may penalize banks even further by requiring that their internal models be reviewed. In circumstances where the internal models are found to be inadequate, banks can be required to adopt the standardized method originally proposed in 1993 by the Basel Accord. The standardized method suffers from several drawbacks, the most noticeable of which is its systematic overestimation of risk, which stems from the assumption of perfect correlation across different risk factors. Overestimating risk leads to higher capital charges, which can have negative impacts on both the profitability and reputation of the bank.

The economic significance of the PS-GARCH model proposed above is highlighted by forecasting VaR thresholds using the PS-GARCH, VARMA-GARCH and CCC models (see Jorion (2000) for a detailed discussion of VaR). In order to simplify the analysis, it is assumed that the portfolio returns are normally distributed, with equal and constant weights. We control for exchange rate risk by converting all prices to a common currency, namely the US Dollar. We use the forecasted variances and correlations from Section 4 to produce VaR forecasts for the period 6 May 1998 to 5 November 2004. The backtesting procedure is used to test the soundness of the models by comparing the realised and forecasted losses (see Basel Committee (1988, 1995, 1996) for further details).

Figures 5-7 show the VaR forecasts and realized returns for each model considered. Both the CCC and PS-GARCH VaR forecasts violate the thresholds 7 times from a possible 1720 forecasts, while the VARMA-GARCH model leads to 6 violations from 1720 forecasts.

Table 3 shows that the mean daily capital charge, which is a function of both the penalty and the forecasted VaR, implied by PS-GARCH is the largest at 10.70%, followed by VARMA-GARCH at 9.76% and CCC at 9.67%. A high capital charge is undesirable, other things being equal, as it reduces profitability. Table 14 also shows that CCC leads to violations that are greater in terms of mean absolute deviations than the VARMA-GARCH and PS-GARCH models. This is particularly important because large violations may lead to bank failures, as the capital requirements implied by the VaR threshold forecasts may be insufficient to cover the realized losses. Finally, CCC also leads to the largest maximum violation.

6. CONCLUSION

The need to create volatility models that can be used to estimate large covariance matrices has become especially relevant following the 1995 amendment to the Basel Accord, whereby banks are permitted to use internal models to calculate their VaR thresholds. While the amendment was designed to reward institutions with superior risk management systems, a backtesting procedure, whereby the realized returns are compared with the VaR forecasts, was introduced to assess the quality of the internal models. Banks using models that lead to a greater number of violations than can reasonably be expected, given the confidence level, are penalized by having to hold higher levels of capital. The imposition of penalties is severe as it has an impact on the profitability of the bank directly through higher capital charges, may damage the bank's reputation, and may also lead to the imposition of a more stringent external model to forecast the VaR thresholds

This paper examined various conditional volatility models for purposes of forecasting financial volatility and VaR thresholds. Two constant conditional correlation models for estimating the conditional variances and covariances are the CCC model of Bollerslev (1990) and the VARMA-GARCH model of Ling and McAleer (2003). Although the VARMA-GARCH model accommodates spillover effects from the returns shocks of all assets in the portfolio, which are typically estimated to be significantly different from zero, the forecasts of the conditional volatility and VaR thresholds produced by the VARMA-GARCH model are very similar to those produced by the CCC model. Furthermore, the models with spillover effects can be computationally difficult as the number of assets becomes large. The paper also developed a new computationally convenient Portfolio Spillover GARCH (PS-GARCH) model, which allowed spillover effects to be included in a more parsimonious manner. This parsimonious model was found to yield volatility and VaR threshold forecasts that were very similar to those of the CCC and VARMA-GARCH models. Overall, the inclusion of multivariate spillover effects was found not to improve the accuracy of the forecasts significantly relative to models without spillover effects.

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Table 1: Descriptive Statistics for Returns

Statistics	S&P500	FTSE100	CAC40	SMI
Mean	0.033	0.020	0.020	0.036
Median	0.029	0.013	0.043	0.037
Maximum	5.771	8.336	10.356	7.049
Minimum	-5.533	-5.681	-10.251	-9.134
Std. Dev.	1.022	1.067	1.346	1.164
Skewness	-0.018	0.118	0.015	-0.120
Kurtosis	6.160	6.254	7.391	7.044
CoV	31.227	54.520	66.002	32.558
Jarque-Bera	1548.350	1649.464	2988.976	2543.419

Table 2: Basel Accord Penalty Zones

Zone	Number of Violations	Increase in k
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is given for 250 business days.

Table 3: Mean Daily Capital Charge and AD of Violations

Model	Mean Daily Capital Charge	AD of Violations		
		Minimum	Maximum	Mean
CCC	9.685	0.0005	2.12514	0.4979
VARMA-GARCH	9.760	0.0001	1.97354	0.4535
PS-GARCH	10.697	0.0001	1.90225	0.4417

Note: The daily capital charge is given as the negative of (3+k)VaR, where $0 \le k \le 1$ is the penalty. AD is the absolute deviation of the violations.

Figure 1: S&P500 Volatility Forecasts

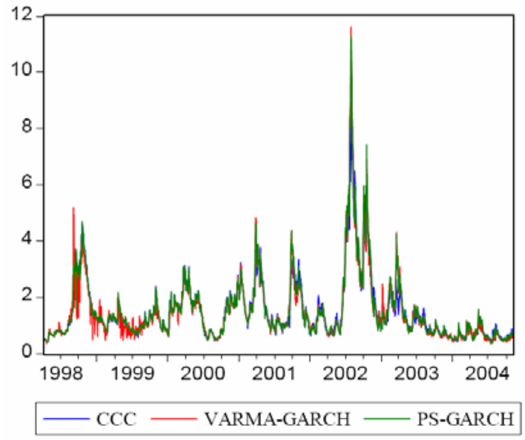

Figure 2: FTSE100 Volatility Forecasts

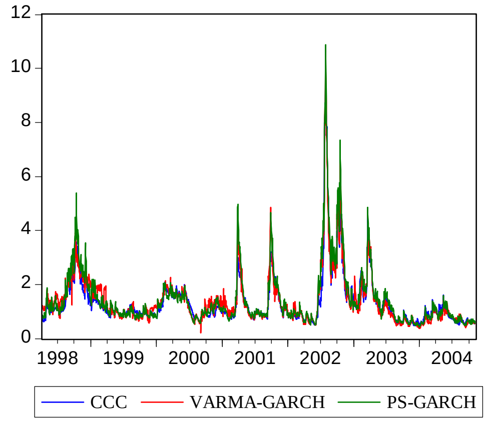

Figure 3: CAC40 Volatility Forecasts

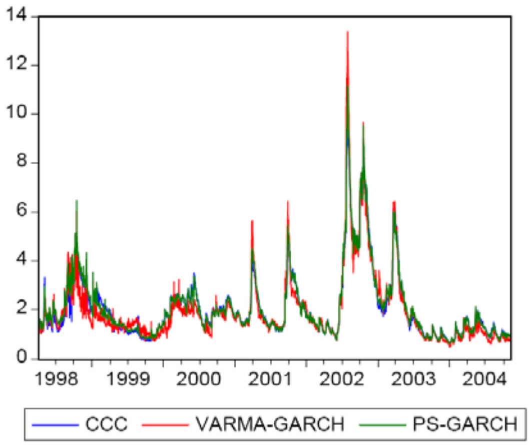

Figure 4: SMI Volatility Forecasts

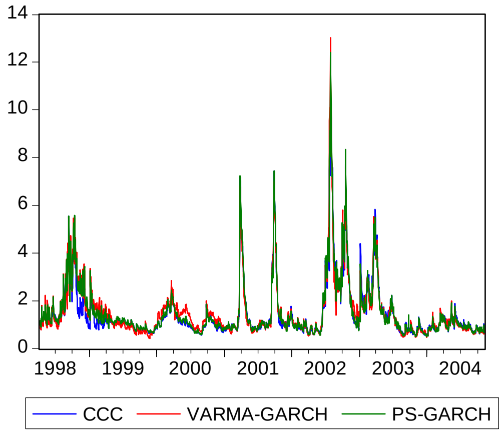

Figure 5: Realized Return and CCC VaR Forecasts

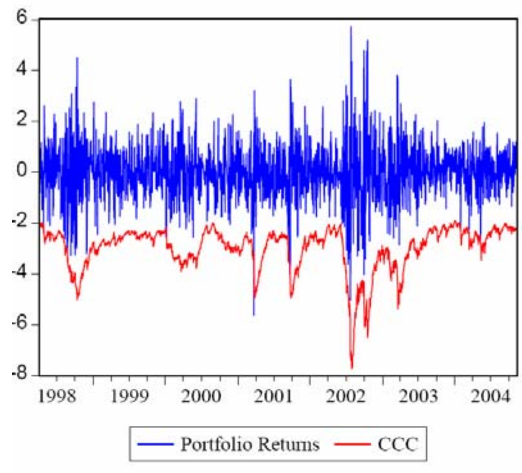

Figure 6: Realized Return and VARMA-GARCH VaR Forecasts

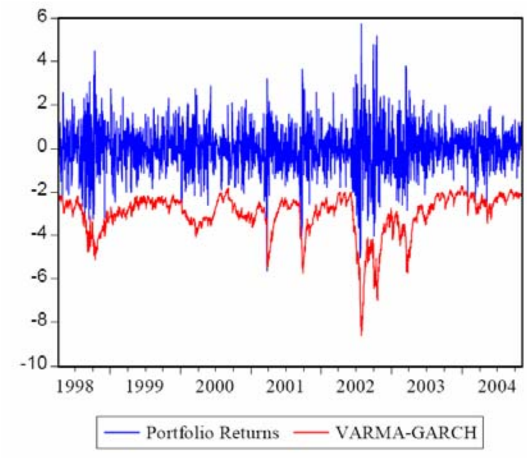

Figure 7: Realized Return and VARMA-GARCH VaR Forecasts